Aggregate Fluctuations and the Industry Structure of the US Economy

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Abstract

Reallocation of inputs in production and substitution across them, are mechanisms through which the economy adjusts to changes in relative efficiency in production across sectors. When input productivity moves along relative prices, cost shares are constant and the input output structure of the economy does not change. I document that cost shares of intermediate inputs fluctuate on average 1.9% in the equipment sector and 1.1% in the consumption sector. Furthermore, cost shares of intermediate inputs from the equipment (consumption) sector are procyclical (countercyclical) across the economy. These facts are used to discipline the behavior of a multisector economy with intermediate goods and neutral and investment specific shocks. I compare the predictions of the model against a comparable economy calibrated to the same steady state, under constant cost shares. I find that neutral shocks become relatively more important in generating output volatility when cost shares are allowed to fluctuate as in the data. Additionally, the paper provides conditions for the existence of a balanced growth path in which all intermediate inputs are used in production, and the economy displays investment specific technological change. [JEL CODES: E23,E22,L6,L7]

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1 Introduction

The input output structure (a summary of the trade in intermediate inputs across sectors) is usually assumed constant in time. However, recent input output data reported at annual frequencies, suggests that the structure changes in time and that those changes are correlated with aggregate activity in the economy.

1. The average absolute change of intermediate inputs' cost shares in the equipment production (consumption goods) sector is 1.9% (1.1%) annually.

2. The off diagonal terms of the input output matrix change more than the diagonal terms(intermediate inputs produced by the same sector).

3. Cost shares of intermediate inputs[1] produced by the equipment (consumption) sector correlate positively (negatively) with aggregate activity.

Changes in the entries of the input-output matrix are a reflection of the pattern of reallocation in the economy in response to changes in relative prices. Relative price change when the efficiency in production of certain sectors improve over others in the economy. Hence, cost share behavior bears information as of the reallocation of factors in response to changes in efficiency in production across sectors, and through it, of the propagation of shocks in the economy. In this paper, I revisit an old question in real business cycle theory: what is the role of sectoral and neutral shocks in aggregate fluctuations? To answer this question I study a two sector economy augmented to allow for intermediate input linkages and consistent with the movements in the input output structure observed in the data. This framework is homomorphic to the canonical model with two sectors, and investment specific and neutral shocks (Greenwood et al. (1997) and Fisher (2006)). The main result is that the augmented economy predicts a stronger

[1] Based on BEA, description of Annual Industry Accounts this include energy, raw materials, semi-finished goods, and purchased services

role for neutral shocks in the volatility of aggregate output vis a vis a comparable economy with a fixed input output structure. Also, the amplification of sectoral shocks is stronger in the flexible cost share economy than in the canonical one.

After the work of Greenwood et al. (1997) we have seen the development of a fruitful research agenda that studies the role of investment specific versus neutral shocks in long run growth and aggregate cyclical fluctuations (See Foerster et al. (2008) and Atalay (2014)). I augment that economy to allow for intermediate good linkages across sectors. The consumption sector produces final and intermediate goods out of capital and intermediate goods from the equipment and consumption sector. In the equipment sector, there are two subsectors. One produces investment goods out of capital and intermediate goods, and the second on produces intermediate equipment goods out of capital and intermediate consumption goods. Capital is sector specific and the stock of capital is fixed at the beginning of each period, before shocks are realized. Although the structure is richer than the canonical model, I show that under certain conditions on the share of inputs in production, the augmented economy reduces to a two sector economy indistinguishable from the standard economy studied in the literature. Among others, the economy is consistent balanced growth and investment specific technical change[2].

In a one sector model, the share of intermediate goods in production has a role in determining the level of GDP, Jones (2011). But GDP growth depends only on aggregate productivity growth, measured as the change in output not explained by a change in primary inputs (labor and capital). In other words, intermediate inputs are irrelevant in determining aggregate fluctuations. In a multisector neoclassical model instead, the production possibility frontier is a weighted measure of the Solow residuals in each sector, Hulten (1978). The computed solow residuals depend not only on the allocation of primary factors but also on the allocation of intermediate goods across sectors. Hence, aggregate output fluctuations are determined by changes in the allocation

[2]While unexplored in this paper, these characteristics are key if the framework is to be used in the empirical analysis of the role of neutral and investment specific technical change through long run restrictions as in Fisher (2006)

of intermediate goods across sectors in response to changes in relative prices.

If we assume that markets are competitive, cost shares of inputs are equal to the elasticity of inputs in production, i.e.

$$Csh^{iJ} = \frac{p^i M^i}{p^J Y^J} = \frac{\partial Y^J}{\partial M^i} \frac{M^i}{Y^J} = \left| \varepsilon^{iJ} \right|$$

where p index prices, Y gross output and M intermediate good consumption. In a frictionless economy, changes in relative prices reflect changes in relative productivity across sectors. When input productivity moves along relative prices one to one, cost shares are constant and the input output structure of the economy does not change. Substitution towards more productively produced inputs may generate output increases in the sector producing the intermediate good as well as in the one consuming it. Cost shares can go up or down. Hence, cost shares movements, jointly with the behavior of relative prices bring us direct evidence of the pattern of reallocation.

Why do these patterns imply different roles for sector specific and neutral shocks? To illustrate, let's think of an economy with three sectors. Two sectors produce intermediate goods out of a linear technology in sectoral productivity. The third sector combines these two inputs to generate the consumption good in the economy using a Leontief technology.

$$Y = \min \left\{ aM^1, M^2 \right\} \qquad M^1 = A^1,\, M^2 = A^2$$

where A^i is exogenous. The equilibrium price of output satisfies

$$p = p^2 + \frac{1}{a} p^1$$

Suppose that productivity improves in sector 2. Then $\Delta A^2 > 0$, and the cost share for input 2 drops as $\Delta p^2 < 0$. The cost share of input 2 is just the relative price of input to output. Total value added does not change because $Y = aA^1$, but aggregate productivity improves as $\Delta TFP = \frac{A^2}{aM^1}\Delta A^2$. Hence, a purely sectoral shock has no impact on aggregate output but improves productivity. A neutral shock (that raises both A^1 and A^2) improves both aggregate productivity and output. Furthermore, should the economist analyzing the economy had imposed a constant cost share structure, it would have predicted an increase in aggregate output after the shock. Substitution towards the now more efficiently produced input would have induced an increase in output of $\Delta Y = Csh_{2Y}\Delta A^2$.

While in the example the disparity in cost share behavior is fully characterized by the underlying production function describing output in each sector, there are many other mechanisms for which cost shares may change differently across sectors, even when operating the same technology. Input specificity is one of them. When looking at aggregate sectoral data, many goods are bundled together. Movements in cost shares may reflect the inability to easily switch across goods that are close together (belong to the same 3 digit NAICS code) but not necessarily the same. Another potential source of cost share fluctuations is the presence of inventories. While inventories should be accounted independently of intermediate goods, data measurements may include items that we would consider inventories. A similar argument follows for equipment parts, which should be accounted as part of investment goods. If a firm has stock up enough intermediate inputs for production within a year of production, changes in relative prices anytime during the year will not be reflected in its input intake. In this paper, I assume movements in costs shares are generated by differences in production technologies only. This allows me to assess the quantitative impact of changes in cost shares while keeping a structure that is very close to the canonical two sector model in the literature.

In the paper, conditions are provided for the existence of a balanced growth path in which all inputs are used in production, yet productivity growth rates in the equipment and consumption sector may differ. When the detrended economy is calibrated to predict the patterns of cost share movements observed in the data, the contribution of neutral shocks to output volatility increases relative to a comparable economy with constant cost shares. In other words, the variance decomposition of an economy calibrated to the same steady state in which constant cost shares are imposed (Cobb-Douglas technologies), indicates that neutral shocks contribute 8% less to aggregate output volatility than they do in a flexible cost share economy. Aggregate output impulse responses to persistent and fully temporary shocks depend on the underlying reallocation patterns embedded in the economy.

Finally, the impact of sectoral shocks is amplified in the flexible cost share economy versus the constant one. In other words, to generate the same volatility in aggregate GDP, and gross output in the consumption and equipment sector, the identified size of the shocks in the economy with a fixed input output structure is larger, than that in the flexible cost share economy.

The rest of the paper is organized as follows. Section 2 described the related literature, Section 3 documents the main finding in the data. Section 4 describes the model and the characterization of the BGP. Section 4 presents the calibration and quantitative results. Section 5 concludes.

2 Literature Review

The literature on the role of sectoral shocks is extensive. The seminal work by Hulten (1978) paved the way for the study of the role of input output linkages in the transmission of sectoral shocks. While the authors find a substantial role for sectoral shocks in shaping aggregate fluctuations in output, much discussion has been triggered since on the plausibility of transmission of idiosyncratic shocks to the aggregate economy. At the heart of the arguments is whether law of large arguments apply to the units that we define as sectors[3].

[3]Dupor (1999) shows that when the network that describes the input output structure is a balanced one, sectoral shocks indeed do not affect aggregates. We have learn much about the characteristics

There are quantitative approaches that exploit the factor structure of a model with input output linkages as in Long and Plosser (1983). The work of Foerster et al. (2008) show that the role of sectoral shocks in explaining aggregate volatility has increased (in relative terms) after the great moderation. Key to the econometric strategy of the paper is the assumption that the input output structure is stable. This paper departs fundamentally from it by allowing trade intensities in intermediate goods to change across time. More recently, Atalay (2014) has used intermediate input purchases to identify the relative importance of industry-specific shocks. In his framework he estimates an elasticity of substitution between value added and intermediate goods different than one. In this paper, I assume the elasticity is between intermediates and value added unitary so that while sector productivity trends may differ across sectors, the economy is consistent with a balance growth path in which all intermediate goods are used in production.

After the work of Greenwood et al. (1997), the analysis of economies with neutral and investment specific shocks, is the preferred choice in the literature studying business cycles. Both the consistency with long run growth and a trend a in the relative price of equipment to consumption is key in a two sector economy like the one I study in this paper. While the papers described earlier allow for a large degree of heterogeneity across sectors in the economy, I keep the economy as close to the now plain vanilla business cycle model as possible. By doing this, I can uncover the role of the input output structure, while 1) providing a flexible framework that a) can be enriched to analyze a richer shocks structure as in Smets and Wouters (2007), b) can accommodate stochastic trends between the investment and consumption sector (as in Schmitt-Grohe

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of the network structure since. Horvath (2000) shows that when the input output structure is sparse (as is the case in the data) sectoral shocks do not fade away in the aggregate. Alternatively, Carvalho and Gabaix (2013) show that when the role of sectors in the economy is unbalanced, in the sense that a few sectors account for most of the value added in the economy, the law of large numbers fails and sectoral shocks can have aggregate impact. Along the same line are the network results by Acemoglu et al. (2012) and Oberfield (2011). Hence, there is nowadays consensus that sectoral shocks can be transmitted to the aggregate economy and have quantitative impact.

and Uribe (2011)) 2) paving the way for future research on the its implications for long run identification strategies as in the seminal work of Gali (1999), augmented later to allow for investment specific shocks in Fisher (2006).

There is an extensive literature in business cycles studying the impact of investment specific and neutral shocks for aggregate fluctuations. Justiniano et al. (2010) show that in a full DSGE model with price markup shocks, neutral technology shocks, Calvo pricing, wage markups shocks, preference shocks, and investment shocks, most of the variability of output is explained by shocks to the marginal efficiency with which final goods are transformed into capital. The structure of intermediate goods trade across sectors is abstracted away. In this paper, the economy is strip out from the rich shock structure and augmented to allow for intersectoral trade in intermediates. This allows me to highlight the relevance of modeling the input output structure with endogenous fluctuations vis as vis an economy with a fixed input output structure. It is shown that the amplification of sectoral shocks is stronger in this economy, than in a comparable economy with a constant input output structure. The particular modelling strategy in which the equipment production sector is split into an investment good producing sector and an intermediate good producing sector allows me to identify the differential impact of shocks to each of these activities. Shocks to the production of investment and intermediates goods are in nature disparate and are shown to have distinct relative impact in output volatility.

Finally, there is an incipient literature applying notions of networks to understand the generation of trade linkages between firms (Oberfield (2013) and Carvalho and Voigtlander (2014)). While the focus of the analysis is different from the one in this paper, both are complementary to each other. Is through the coordinated behavior of all those firms that the decision on trade linkages matters for the aggregate dynamic of the economy. In this paper I show that cost shares of fairly aggregated sectors fluctuate in time and they are relevant in understanding the role of investment specific and neutral shocks in the economy. It remains to be shown that the fluctuations in aggregate cost shares are consistent with the firm behavior observed in the data.

3 Empirical Facts

3.1 Input-Output structure

I study make-use tables from 1993 to 2012 as reported by the Bureau of Labor Statistics based on BEA data. The series are presented for 195 sectors, and values are current US dollars. The data Appendix describes in detail the sectors that have been included in the analysis.

The objective of this analysis is to describe the changes in the input output structure vis a vis the aggregate level of activity in economy. The level of aggregation across sectors is key in producing the facts documented in this section. For the purpose of the analysis in an economy with two sectors as the one presented in the body of the paper, aggregation of consumption and equipment/investment sectors is enough. However, in the analysis of the empirical facts I present results where I aggregate sectors to build an Input Output matrix with 33 industries, consistent with the KLEMS sectoral data available at BEA. Then I further classify these 33 sectors as investment/equipment, consumption sector, agricultural and mining sector.

Independent of the level of aggregation, the investment sector is constructed to include equipment producing sectors consistent with the analysis in Cummins and Violante (2002). In other words, the aggregation rule is consistent with the construction of relative price indexes that are used to describe the path of investment specific technical change.

In the analysis I abstract from the behavior of agricultural and mining sectors for several reasons. First, the assumption of constant returns to scale in technology that I use later in the model economy is unlikely to hold in these sectors, where there are large fixed costs of operation. Second and most important, fluctuations in price of these commodities may not be tight to changes in relative productivity vis a vis other sectors in the economy, but rather to developments in international commodity markets. The government, except postal services, has been abstracted away from this analysis.

The cost share of input i in sector j is defined as

$$Csh^{iJ} = \frac{p^i M^{iJ}}{p^J Y^J}$$

where Y^J denotes gross output in sector j, M^{iJ} is the intermediate good i intake of sector j and p denote prices. Hence, cost shares fluctuate whenever changes in relative prices are not fully translated into changes in input productivity $(\frac{Y^J}{M^{iJ}})$.

Figures 1 to 6 display time series of cost shares of consumption and equipment intermediate goods for various sectorial aggregations. The period of analysis includes the Great Recession, where the input output structure experienced a substantial shake out. The nature of those changes are out of the scope of this particular analysis. However, the panels include a line identifying the date collapse in the US financial market. This panel are constructed singling out the behavior of particular sectors that might be driving the behavior of the aggregate cost shares of equipment and consumption. In the first two panels I present data with all sectors are described in the baseline classification in the appendix. In the next two panels, I single out the behavior of the construction sector, potentially important in the developments of the late 90's and 2000's up to the recession in 2007. I also single out Utilities. In the last two panels I show the behavior of cost shares of the finance sector and the real state sector. In each of the last 4 panels,the cost shares of consumption and equipment have the corresponding sector under analysis, substracted out.

In Figures 1 and 2 I show the behavior of cost shares of each type of intermediate goods in the consumption and equipment sector, and for completeness, in the agricultural and mining sector. Own cost shares fluctuate substantially (consumption in consumption, and equipment in equipment). These sectors aggregate up changes in relative prices across subsectors, so it is possible for the within sector cost share to fluctuate with movements in relative prices and not only with input productivity. The cost share of consumption intermediates in equipment displays a slight upward trend up to 2000s that reverts in the next decade. The cost share of equipment intermediates in equipment displays a mirror dynamic, with the cost share dropping up to the 2000s and increasing later on. The cost share of equipment in consumption displays a downward trend that may well be explained by the drop in the relative price of equipment to consumption goods. If we normalize cost shares to account only for those sectors accounted in the consumption and equipment sectors[4], the cost share of consumption in the equipment sector averages 43% in the sample period, while the cost share of equipment intermediates in consumption averages 7% and displays a declining trend. If only these two aggregate sectors are considered in the sample, the cost share structure is as depicted in Table (1) .In the sample period, gross output of the consumption sector is four times larger than gross output of the equipment sector. Also, in the consumption(equipment) sector 67% (48%) of gross output is used as intermediate input in other sectors. The rest is either consumed or adds to the capital stock.

Input/Sector	Equipment	Consumption
Equipment	43%	7%
Consumption	57%	93%

Table 1: Share of Service and Manufacturing Inputs (intermediate goods from the Agriculture and Mining sector have been factored out)

In Figures 3 and 4 I disaggregate the behavior of the construction and utilities sector from the overall consumption sector. The exercise is designed to understand if some of

[4]This normalization is consistent with the model presented in the next section, in which only this two sectors are accounted for.

the dynamic described before are explained by particular sectors. When construction and utilities are abstracted away the consumption sector, the cost share of equipment in consumption displays a less steep downward trend than for the full sector, in particular after 2000.The cost shares of i equipment intermediate inputs in construction is relatively stable. The cost share of consumption intermediate drops substantially up to 2000 and raises thereafter. Utilities cost shares of equipment and consumption intermediate goods behave as mirror images. The dynamic of the cost share in of consumption in consumption is very similar to the one depicted in the previous panel. This indicates that neither construction nor utilities explain the decrease in cost share of consumption intermediates in consumption from the 2000s onwards.

Figures 5 and 6 display a disaggregation for the finance sector and real state sector. The share of consumption intermediates in the consumption sector displays a downward trend for the whole sample period, indicating that part of the raise in the early 2000 are explained by the consumption intermediate share in the finance sector, which remains relatively stable. Noticeably, the cost share of consumption in equipment is much more volatile when the finance and real state sector are abstracted from consumption.

In summary, cost shares fluctuate substantially. The cost share of equipment in consumption sectors displays a downward trend, while the remaining cost shares are relatively stable. Cost shares of consumption intermediates went up on average up to the beginning of the 2000s and then down to the end of the sample period. The cost share of equipment in equipment sectors behaved as mirror image of that pattern.

Next I would like to describe the year on year changes in cost shares. In other words, I would like to describe changes in (1). To study variation across inputs I compute average absolute changes year on year. Those are presented in table (2)

Input/Sector	Equipment	Consumption
Equipment	1.6%	2.9%
Consumption	2.2%	0.9%

Table 2: Yearly average absolute change

The off diagonal terms, are larger than the diagonal terms, and in particular, the largest movements are for equipment intermediates in the consumption sector. Relative price changes within a category, i.e. computers and transportation equipment, are aggregated out in the changes reported in the diagonal of the table. Changes in the relative price of consumption and equipment basket are reflected in the fluctuations in the off diagonal terms. To grasp the magnitude of these changes,one could compute the absolute average deviations from the mean share over the sample period, as in table(3)

Input/Sector	Equipment	Consumption
Equipment	2.4%	10.1%
Consumption	3.5%	0.9%

Table 3: Average absolute deviation relative to mean share

Changes in the cost share of equipment in consumption are accounted large (10.1% on average), and those of consumption in the equipment sector average 3% of the mean. However, as depicted in figures 1 to 6, some of the share series contain longer term trends. To avoid imputing changes in cost shares as just shifts along the trend, I also report deviations from an hp-trend. These absolute deviations are reported in (4). Once we account for this trend, deviations in own intermediate inputs cost shares drop below 1% per year. and deviations in the cost share of consumption in equipment and equipment in consumption are 1.3% and 1.5% respectively.

Input/Sector	Equipment	Consumption
Equipment	0.9%	1.5%
Consumption	1.3%	0.4%

Table 4: Average absolute deviation relative to HP- trend (smoothing factor, 6.25)

To summarize, changes in cost shares are not negligible whether accounted as year on year changes or as deviations from trend. Changes in cost shares of intermediate inputs produced by other sectors (the off diagonal terms in the last three tables) are larger than those in the diagonal terms.

The asymmetry in the industry structure depicted in Table(1) and the contribution of each sector in value added and gross output are important features that the model economy needs to capture to assess: a) the elasticity of the cost shares to changes in relative prices, b) the aggregate impact of those changes. Elasticities of substitution across inputs (hence, cost shares) depend on the industry structure as summarized by (1) .However, the relevance of shocks in the aggregate depend on their contributions to value added and gross output.

Finally, let me describe the correlation of cost share changes with aggregate activity. I define value added as the sum of the dollar value of value added in the equipment and consumption sectors. I report correlations for three different time series. The full sample includes the Great Recession (GR), the second sample only focuses on the periods up to the GR. The third sample interpolates the pre and post 2008 values abstracting from the drop in activity in 2008. Table (5) shows that cost shares of consumption intermediate goods are countercyclical irrespective of the sample period. Cost shares of equipment intermediates in the equipment sector are procyclical in the full sample, but acyclical if we consider the period pre 2008 or the interpolated data. The correlation of the cost share of equipment intermediates in the consumption sector with aggregate value added displays the largest disparities across sample periods. While for the full sample the correlation is positive , in the pre 2008 period it is identified negative of about the same magnitude. When we interpolate to abstract from the GR the cost share appears acyclical

	Full Sample		Pre-2008		without GR	
Input/Sector	Eq	Co	Eq	Co	Eq	Co
Equipment (Eq)	0.22	0.10	0.05	-0.14	0.07	-0.08
Consumption (Co)	-0.22	-0.27	-0.13	-0.33	-0.12	-0.23

Table 5: Correlation with Industrial Value Added

For the calibrated exercise at the end of the paper I will use data from the full sample. The particular shifts in the input output structure that occur during the break out of the recession remain to be studied, possibly at a higher level of disaggregation.

The countercyclicality of the cost share of consumption in equipment is important in view of the extensively documented countercyclicality of the price of equipment. Negatively correlated cost shares indicate that input productivity drops less than the drop in the relative price of equipment to consumption. In good times the relative price goes down, inducing the cost share of consumption in equipment to increase if there are no changes in input productivity. For the cost share to drop, input productivity has to increase in the equipment sector.

If we abstract from the changes in relative prices that are certainly occurring within each of these fairly aggregate sectors, the correlations on the diagonal terms of the table indicate that: a) input productivity increases in the consumption sector when aggregate activity booms on average, and b) that input productivity in the equipment sector drops during activity booms on average. Note that if the relative price of one of the categories within a sector is dropping dramatically, say the price of computers relative to transportation equipment, that shift in prices might be reflected as a procyclical cost share for intermediates produced in the same sector.

As mentioned early, these results are not invariant to the degree of aggregation in the economy. In Figures 7 and 8 I compute the correlation of cost shares of equipment and consumption intermediate goods for 33 sectors, and for the full sample of 170 sectors (abstracting the government).

To conclude this section, let me summarize the main features observed in the data:

By studying an economy consistent with these features I will argue that these facts are key in understanding the role of sectoral and neutral shocks as well as the amplification of shocks in the economy.

1. Sectors have disparate roles as input suppliers of other sectors in the economy.

2. The average absolute change of intermediate inputs' cost shares in the equipment production (consumption goods) sector is 1.9% (1.1%) annually.

3. The off diagonal terms of the input output matrix change more than the diagonal terms(intermediate inputs produced by the same sector).

4. Cost shares of intermediate inputs produced by the equipment (consumption) sector correlate positively (negatively) with aggregate activity.

4 Model

4.1 Environment

This is a discrete time, infinite horizon economy.

There are two final goods produced in the economy, equipment and consumption goods. Additionally, intermediate equipment goods are produced.

There are three production sectors in the economy. I assume there is a representative firm in each sector. All markets are competitive and the technologies are constant returns to scale. The diagram 6 displays the input output structure of the economy under analysis. A cross indicates a positive entry in the matrix. To distinguish between goods produced by the equipment sector, I call X_2 the sector producing intermediate equipment and X_1 the sector producing investment goods (capital)[5].

	Sector	Intermediate Demand			Final Demand	
		E		C		
		X_2	X_1	Y	Equipment	Consumption
E	X_2		x	x		
	X_1				x	
C	Y	x	x	x		x
	Capital	x	x	x		

Table 6: Input Output matrix of the economy

Total value added in this economy equalizes the sum across entries in the last two columns (GDP). Gross output per sector corresponds to the row sum across all columns. Total cost corresponds to the column sum per sector. The cost share is the ratio between

[5]This distinction is useful for the analysis of the balance growth path in particular. I will show that under certain conditions, this economy reduces to a two sector economy where both sectors produce final and intermediate goods, and it is possible to observe investment specific technical change.

a particular entry in the intermediate demand section and gross output. The cost shares in the model are constructed as in the data, separating out expenses in capital services.

There is a representative household with standard preferences over final consumption goods. She maximizes lifetime utility by choosing a consumption stream as well as purchases of investment goods.

4.2 Representative Household

The representative household maximizes lifetime utility subject to its budget constraint. Her income stems from the rental of capital to the firms in the economy and from claims to the profits of those firms. Note that capital is specific to a sector and hence capital cannot be instantaneously reallocated from one sector to another. The non-negativity constraint in investment goods should hold for each capital type.

$$\max_{c_t^s, c_t^m, x_t} \sum_{t=0}^{\infty} \beta^t \; U(c_t)$$

subject to

$$p_t^y c_t + p_t^{x_1} x_1 \leq r_t \left(k_{x1} + k_{x2} + k_y\right) + \sum_{j=y,x_1,x_2} \pi_t^j$$

$$k_{x1}^{'} - k_{x1}(1-\delta_x) = i_{x1} \qquad (\kappa_x)$$

$$k_{x2}^{'} - k_{x2}(1-\delta_x) = i_{x2} \qquad (\kappa_x)$$

$$k_{y}^{'} - k_{y}(1-\delta_y) = i_{y} \qquad (\kappa_y)$$

$$i_y + i_{x1} + i_{x2} = x_1 \qquad \text{and } i_j \geq 0 \text{ for } j = y, x_1, x_2$$

where β is the discount factor; p^j indexes prices for alternative goods $j = y, x_1$, i.e. final consumption and investment goods, respectively; the capital stock is k_t^j with rental rate r_t and the profits of the firms in each sector are π_t^j. The depreciation rate is allowed to differ between equipment production sectors and consumption production sectors.

4.3 Consumption Goods Sector

The representative firm in the consumption sector maximizes profits each period. It has available a technology that uses intermediate goods (M) and capital goods. Once the productivity of all sectors is realized, it chooses its input purchases. The problem of the firm reads

$$\max_{M_t^{yy}, M_t^{xy}, k_t^y} \; p_t^y Y_t^y - p_t^y M_t^{yy} - p_t^{x_2} M_t^{xy} - r_t k_t^y$$

subject to

$$Y_t = \exp(A_t^g)\,(k_t^y)^{\alpha_{y1}}\left(\alpha_{y2}\,(M_t^{yy})^{\rho_y} + (1-\alpha_{y2})\,(M_t^{xy})^{\rho_y}\right)^{\frac{\alpha_{my}}{\rho_y}}$$

where A_t^g is a Hicks Neutral productivity shock. For simplicity we have assumed that shocks to the productivity of the consumption good sector correspond to aggregate shocks[6]. The intermediate good purchases from sector j are M_t^{jy}; k_t^y is the stock of capital used in production, α^{y1} is the share of the capital/value added in gross output; $(1-\rho_y)^{-1} \in (-\infty, 1)$ is the elasticity of substitution across intermediate goods in the equipment sector; α_{my} is the share of intermediates in value added; and α_{y2} corresponds to the share of consumption intermediate inputs in the production of consumption goods when $\rho_y = 0$ (Cobb-Douglas technology).

Labor is assumed away in this analysis. Or in other words, one could assume that capital and labor are one to one in production, with an elasticity of substitution equal to zero, i.e. a Leontieff technology. It might be potentially important to model the substitution patterns between labor and capital as in Koh and Santaeulalia-Llopis

[6] Shocks particular to this sector can be incorporated. However, we expect the predictions of that economy to be analogous to this one. The current set up correspond in which any change in idyosincratic productivity in this sector is reflected in changes in the relative productivity of the other two sectors in the economy. Quantitatively, the modeling strategy may make a difference in the variance decomposition exercise, so robustness checks will be run.

(2014). This substitution patterns can be complementary to the shifts in intermediate input intake generated by the model.

4.4 Equipment Sectors

4.4.1 Investment Goods

The representative firm in the equipment investment sector maximizes profits by choosing intermediate good purchases of both consumption and intermediate equipment goods. Its problem reads

$$\max_{M_t^{xx}, M_t^{yx_1}} \; p_t^{x_1} X_t^1 - p_t^{x_2} M_t^{xx_1} - p_t^y M_t^{yx_1}$$

subject to

$$X_t^1 = \exp(A_t^g) \exp(A_t^{x_1}) \left(k_t^{x_1}\right)^{\alpha_{x1}} \left(\alpha_{x2} \left(M_t^{xx_1}\right)^{\rho_x} + (1-\alpha_{x2}) \left(M_t^{yx_1}\right)^{\rho_x}\right)^{\frac{\alpha_{mx}}{\rho_x}}$$

where $A_t^{x_1}$ is a Hicks Neutral sectoral productivity shock, M^{ix} are intermediate good i purchases in sector X_1; $(1-\rho_x)^{-1} \in (-\infty, 1)$ is the elasticity of substitution across intermediate goods in the equipment sector; α_{mx} is the share of intermediates in value added; and α_{x2} corresponds to the share of intermediate equipment inputs in the production of investment goods when $\rho_x = 0$.

4.4.2 Intermediate Goods

The representative firm in this sector maximizes profits by choosing capital and intermediate goods from the consumption sector. Its problem reads

$$\max_{M_t^{yx_2}, k_t^y} \; p_t^{x_2} X_t^2 - p_t^y M_t^{yx_2} - r_t k_t^{x_2}$$

subject to

$$X_t^2 = \exp(A_t^g)\exp(A_t^{x_2})\left(k_t^{x2}\right)^{\zeta}(M_t^{yx_2})^{\alpha_{mx_2}}$$

where $A_t^{x_2}$ is a Hicks Neutral productivity shock, $M_t^{yx_2}$ is the purchase of intermediate consumption goods; k_t^{x2} is the stock of capital used in production; and ς corresponds to the share of capital in the production of intermediate equipment goods and α_{mx_2} is the share of intermediates in value added.

4.5 Productivity

Each sector takes the realization of the productivity process as given. Productivity has two elements, a deterministic trend and a noise term. Let $\mathbf{A}_t \equiv \{A_t^g, A_t^{x_1}, A_t^{x_2}\}$ be the current realization of the shocks in the economy. The dynamic of $\mathbf{A}$ is described as

$$\mathbf{A}_t = \Gamma(\mathbf{A}_{t-1})$$

$$\mathbf{A}_t = (1+\boldsymbol{\gamma}_t)\mathbf{A}_0 + \Lambda_t$$

where $\boldsymbol{\gamma}_t$ is a vector collecting the time trends and Λ_t is the noise in the process, $E(\Lambda_t) = 0$.

The noise term has in turn two elements. One that is purely temporary and I call $\boldsymbol{\epsilon}_t$ and a persistent component $\mathbf{z}_t$ with persistence $\boldsymbol{\theta}$ and innovation $\boldsymbol{\eta}_t$. In other words, the noise structure is

$$\Lambda_t = \mathbf{z}_t + \boldsymbol{\epsilon}_t$$

$$\mathbf{z}_t = \boldsymbol{\theta}\mathbf{z}_{t-1} + \boldsymbol{\eta}_t$$

$$\Lambda_t = \boldsymbol{\theta}\Lambda_{t-1} - \boldsymbol{\theta}\boldsymbol{\epsilon}_{t-1} + \boldsymbol{\eta}_t + \boldsymbol{\epsilon}_t$$

$\boldsymbol{\epsilon}_t \backsim N(0,\Sigma^{\epsilon})$ and $\boldsymbol{\eta}_t \backsim N(0,\Sigma^{\eta})$. The variance covariance matrix of the shocks are Σ^{ϵ} and Σ^{η} independent from each other. Whereas the $\boldsymbol{\epsilon}_t$ shocks are purely temporary, the time series of gross output and value added at the sector level will display persistence, through the impact of these shocks on the accumulation of sector specific capital. Also, whereas both persistent and purely temporary shocks may be independent across sectors, the economy will display comovement due to the intermediate input linkages.

5 Equilibrium

Before defining the equilibrium let me introduce some additional notation. Let $\mathbf{p}_t \equiv \{p_t^y, p_t^{x_1}, p_t^{x_2}\}$ be the vector of prices in the economy, $\mathbf{M}_t^y \equiv \{M_t^{yx_1}, M_t^{yx_2}, M_t^{yy}\}$ be the vector of intermediate consumption goods, $\mathbf{M}_t^x \equiv \{M_t^{xx_1}, M_t^{xy}\}$ be the vector of intermediate equipment goods.

Definition 1 *A competitive equilibrium is an allocation of consumption,investment and capital* $\left\{c_t, \{i_t^j, k_{t+1}^j\}_{j=y,x_1,x_2}\right\}_{t=0}^{\infty}$, *as well as intermediate good consumption* $\{\mathbf{M}_t^y, \mathbf{M}_t^x\}_{t=0}^{\infty}$, *such that given a system of prices,* $\{\mathbf{p}(\mathbf{A}_t), r(\mathbf{A}_t)\}_{t=0}^{\infty}$, *the exogenous dynamic for sectoral productivity* $\mathbf{A}_{t+1} = \Gamma(\mathbf{A}_t)$ *and the initial stock of capital* k_0^j,

1. *The representative household maximizes utility*

2. *The representative firm in each sector maximizes profits*

3. *Markets clear:*

   (a) $c_t + M_t^{yy} + M_t^{yx_1} + M_t^{yx_2} = Y_t$

   (b) $i_t^y + i_t^{x_1} + i_t^{x2} = X_t^1$

   (c) $M_t^{xx_1} + M_t^{xy} = X_t^2$

I now describe how the production possibility frontier changes in this multisector economy. As in Hulten (1978), the PPF is a weighted average of Solow residuals of different sectors in the economy. Let $\widetilde{x}_t$ be the log deviation of variable x from its steady state value. Deviations in aggregate efficiency in period t, can be described by

$$\widetilde{T}_t = \sum_J \frac{p_t^J Y_t^J}{\sum_j p_t^j \left(Y_t^j \gamma_t^j\right)} \widetilde{Z}_t^J \tag{1}$$

Fluctuations in the solow residual in each sector are characterized by

$$\widetilde{Z}_t^y = \widetilde{Y}_t - \sum_{i=1}^{S} Csh^{iy} \widetilde{M}_t^{iy} - Csh^{ky} \widetilde{k}_t^y \tag{2}$$

$$\widetilde{Z}_t^{xj} = \widetilde{X}_t^j - \sum_{i=1}^{S} Csh^{ix} \widetilde{M}_t^{ix} - Csh^{kx} \widetilde{k}_t^{xj} \tag{3}$$

Hence, the residual is the change in aggregate output not explained by changes in the input of production. Using the definition of cost shares, movements in intermediate input intake per sector can be characterized by

$$\widetilde{M}_t^{iJ} = \widetilde{Csh}_t^{iJ} + \widetilde{p}_t^J - \widetilde{p}_t^i + \widetilde{Y}_t^J$$

Hence, whether equilibrium prices adjust so that the log deviation in intermediate purchases is the same in the constant and flexible cost share economy should be assessed in the context of a general equilibrium model. There is no reason to believe this will be the case. If predicted log deviations in intermediate purchases differ across economies, so will productivity and through them, predicted aggregate productivity changes.

6 Balanced Growth Path

Before moving on to the quantitative assessment of the model, I describe the balance growth path of the economy and the conditions that reduce this economy to a plain vanilla two sector economy as in Greenwood et al. (1997).

A Balance Growth Path is a path of gross output in the consumption, aggregate consumption, intermediate inputs intake, and equipment intermediate sector gross output such that they all grow at a constant equal rate, and a path of aggregate investment, capital and investment at the sectoral level, such they also grow at a equal constant rate, possibly different from the one of aggregate consumption.

Theorem 1 *Given the technologies assumed for this economy, a Balanced Growth Path (BGP) where all intermediate goods are used in production exists iff technology is Cobb Douglas in capital and intermediates and either 1) the elasticity of substitution across intermediate goods, equals unity (Cobb-Douglas technology); or 2) there are no linkages through intermediate goods between the equipment and consumption sector; 3) productivity growth in the consumption sector and intermediate equipment sector are proportional by a factor* $(g^x)^{\alpha_{y1}-\zeta}(g^y)^{\alpha_{my}-\alpha_{mx_2}}$ *,where* g^x *is the growth rate of gross output in the investment sector and* g^y *is the one in the consumption sector.*

The first result is analogous to that in Ngai and Pissarides (2007) in an economy with structural change. The second result is well know as it reduces the economy to one like the one in Greenwood et al. (1997). The third one allows me to study the economy that has been described in the previous section. One with non-trivial heterogeneous productivity processes across different sectors while allowing for fluctuations in cost shares. It is worth mentioning that while cost shares are allowed to change, in equilibrium they will be constant along the BGP. In the third case, productivity growth in the intermediate equipment and consumption sector are allowed to differ iff the shares of capital and intermediates in value added are different across sectors. If productivity growth rates are proportional, then output in the intermediate equipment sector and in the output sector grow at the same rate.

Corollary 1 *For a given productivity growth rate in the investment sector, it is possible to find a set of parameters (share of capital and intermediates) such that productivity growth in the intermediate equipment sector equalizes the one in the investment sector* *and the BGP is preserved. Productivity gains should satisfy*

$$\gamma^x = \gamma^{x_2} = (\gamma^y)^{\frac{1+\psi_{x2}\left(\alpha_{y1}-\zeta\right)+\psi_{y2}(\alpha_{my}-\alpha_{mx_2})}{1-\psi_{x1}\left(\alpha_{y1}-\zeta\right)+\psi_{y1}(\alpha_{my}-\alpha_{mx_2})}}$$

Hence, if the previous relationship is satisfied between productivity gains in the investment and the consumption sector, the economy resembles an economy with two sectors in which the only shocks are a neutral and an investment specific one.

It is worth describing equilibrium growth rates for the case that will be analyzed in the rest of the paper (3). Growth rates of gross output along the BGP are convex combinations of the productivity growth in the investment and consumption sector.

$$g^x = (\gamma^x)^{\psi_{x1}} (\gamma^y)^{\psi_{x2}}$$

$$g^y = (\gamma^x)^{\psi_{y1}} (\gamma^y)^{\psi_{y2}} = g^{x_2}$$

$\psi_{x1} = \frac{1-\alpha_{my}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{x2} = \frac{a_{mx}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$. Also, $\psi_{y1} = \frac{a_{y_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{y2} = \frac{1-a_{x_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$.

In an economy with constant returns to scale and no labor, so that $a_{y_1} + a_{my} = 1$ and $a_{x_1} + a_{mx} = 1$, the growth rates of output are identical across sectors. In an economy where $a_{y_1} + a_{my} < 1$ and $a_{x_1} + a_{mx} = 1$, the consumption sector gross output will grow slower than the investment sector. If instead, $a_{y_1} + a_{my} = 1$ and $a_{x_1} + a_{mx} < 1$, the consumption sector will grow faster than the investment equipment sector. In other words, whenever the share of intermediates in production in the equipment (consumption) sector is relatively small, the consumption sector grows faster (slower) than the investment equipment sector.

7 Quantitative Exercises

The quantitative strategy is as follows. First the model is detrended using the BGP results from the previous section. In particular, the following transformation of variables generates a stationary economy

$$\frac{Y_t}{Q_t^y}, \frac{X_t^2}{Q_t^y}, \frac{M_t^{ij}}{Q_t^y}, \frac{C_t}{Q_t^y}, \frac{X_t^1}{Q_t^x}, \frac{k_t^i}{Q_t^x}, \frac{i_t^i}{Q_t^x}$$

where

$$Q_t^x = \left(A^{x_1}\right)^{\psi_{x1}} \left(A^y\right)^{\psi_{x2}} \quad \text{and} \quad Q_t^y = \left(A^{x_1}\right)^{\psi_{y1}} \left(A^y\right)^{\psi_{y2}}$$

Second, I calibrate the model to match the steady state behavior of the industry structure (i.e. the share of intermediate inputs in each sector), the volatility of gross output and value added. To match the cyclical behavior of cost shares observed in the data, I calibrate the variance covariance matrix of the shock structure as well as the elasticities of substitution across intermediate goods.

Third, I calibrate a comparable economy with Cobb-Douglas technologies (constant cost shares) to generate the same steady state of the baseline economy.

With these two economies I run alternative experiments. First, I compute impulse responses for identical shocks to test the propagation properties of each economy. Second, I simulate each economy and compute a variance decomposition of the generated path for output and aggregate TFP, for neutral and investment specific shocks.

I have used data from the Capital Flow Table of 1997 to compute investment levels across sectors. Capital stocks for the same year across sectors were obtained from the EUKLEMS database. Nominal shares of intermediate inputs were obtained from annual Input Output tables at chained dollars of 2005 as reported by BLS. The relative price of equipment to consumption good was obtained as averages of quarterly data as reported in DiCecio (2009), computed following Cummins and Violante (2002) methodology.

For these exercises, I assume that the share of intermediates in value added is the residual after deducting the share of capital to assure constant returns in each sector. Under such specification, growth rates of gross output are the same across all sectors (as explicit in the definition of ψ_{x_i}, ψ_{y_i} whenever $\alpha_{j1} = (1 - \alpha_{mj})$)

7.1 Calibration

The model is calibrated to annual frequencies mainly because the data on intermediate good cost shares is available at that frequency. Table 7 describes the set of parameters that were set independently of the model conditions. The discount factor was set consistent with a annual interest rate of 2%. The capital depreciation was set to 5% per year (as in Cooley and Prescott, 1995). I also need to calibrate the growth rate along the BGP. The trends are obtained as gross output weighted average sector growth rates by KLEMS. These are computed 1978 to 2007[7]. Because there is no labor in the economy, the model is bounded to generate the same growth trend in the equipment and consumption sector. In the baseline calibration I use the growth rate for the equipment sector equal to 3.15% per year. I later draw sensitivity analysis assuming instead the average growth rate observed in the consumption sectors, 1.5%.

From the optimality conditions for capital (x^* corresponds to the steady state value of variable x) we obtain,

$$\frac{1 + g^x - \beta(1-\delta)}{\beta} \frac{k^*_{x_1}}{X^*_1} = \alpha_{x1}$$

Hence, I need either a measure of capital output ratio in the investment sector, or a measure of capital capital services in gross output, α_{x1}. I use the latter. The feasibility condition that dictates that gross output in sector X_1 corresponds to total investment in the economy as reported in the Flow of Funds. Capital services are obtained from KLEMS data.

[7]If the average is computed over a time frame comparable to the input output data, the growth rate in the equipment sector raises 1% and in the consumption sector raises 0.2%. Productivity growth for these period is 5.99% in the equipment sector and 1.5% in the consumption sector (value added measures of TFP).

Following a similar strategy we can calibrate the share of capital in the consumption sector as

$$\frac{1+g^x-\beta(1-\delta)}{\beta}\frac{\lambda_x^* k_y^*}{\lambda_y^* Y^*}=\alpha_{y1}$$

Consistent with the literature (Hornstein and Praschnik (1997) and Huffman and Wynne (1999)), the calibrated share of capital in the consumption sector is slightly higher than the one in the investment sector.

We are left to calibrate, two elasticities of substitution ($\frac{1}{1-\rho_x}$, $\frac{1}{1-\rho_y}$) and the shares of input in production, as well as the variance covariance matrix of the shocks. I calibrate them jointly matching moments of the data. The moments targeted are twelve, described in Table 9. They include the industry structure (the cost shares reported in Table (1)); the correlation of cost shares to aggregate value added (reported for the pre Great Recession period in Table (5)); the volatility of aggregate GDP, and the volatility of gross output in each sector as well as the persistence of the cyclical component (hp filtered) of each series.

Table 8 describes the set of calibrated parameters. The first two parameters are closely tight to the share of intermediates from each sector. The elasticities of substitution across inputs and the full structure of shocks are identified through the correlations of cost shares with aggregate activity, as well as the persistence of shocks and volatility of the aggregate series. The persistence parameters θ where taken as primitive in the simulations, and several sensitivity analysis done over the values of the primitive. The calibrated persistence and volatility of the aggregate series are particularly sensitive to the underlying persistence of the shock in the investment sector θ_x. I choose ς to assure that the steady state of the model is well defined, i.e. there is a set of non-negative prices that solve the allocation.

In the preferred calibration, the share of consumption intermediate across sectors is similar in the investment and consumption sectors. The elasticities of substitution across intermediate goods are identified less than one in both sectors. The elasticity of substitution is higher in the consumption sector. The shock structure is such that the volatility of neutral shocks, temporary and persistent are always lower than the volatility in the equipment sector. Among equipment producing sectors, we identify higher (lower) volatility for purely temporary(persistent) shocks in the production of intermediates, than in the investment production sector. Finally the covariance of temporary shocks in the equipment production sector is identified negative at -0.72. Intermediate equipment goods are typically parts and unfinished goods that would eventually contribute to the stock of capital in the economy. When the final equipment sector entails a positive shock capital goods turn cheaper vis a vis intermediate equipment goods. Output in the equipment sector raises and the relative price of final to intermediate equipment goods drops. In the data, we have identified that cost shares in the equipment sector are procyclical. Hence, in input productivity in the equipment final goods sector raises, it may possible offset the impact of the change in relative prices, inducing countercyclical cost shares which is at odds with the data (recall that the elasticity of substitution across intermediate inputs in the investment sector is close to one).

With this calibration the model is able to generate an industry structure where the cost shares of consumption and equipment in the equipment sector are very close to the ones observed in the data (the cost share of consumption goods in equipment is 57% and the model predicts it to be 53%). The model however generates a cost share of consumption goods in the consumption sector 10% lower than in the data(the cost share of consumption goods is 81% in the model and 93% in the data).

In terms of the correlations of cost share movements with aggregate value added, the model replicates the countercyclicality of consumption sector cost shares observed in the data. Almost by construction however, it is not able to generate the disparity in correlations across different types of intermediate inputs. The size of the correlation is slightly overestimated for the cost share of equipment and consumption intermediates in equipment. The model predicts a lower correlation of cost shares in the consumption sector. The predicted correlation is closer in magnitude to the one documented in the data for the cost share of equipment in consumption. The model underestimates the correlation of the cost share of consumption in consumption. Correlations between aggregate output and cost shares increase when the size of the shocks, in particular the neutral shock, increases. However, higher volatility of the neutral shocks implies a much larger volatility of output than observed in the data.

Regarding the volatility of output, the statistic in the data is slightly higher than predicted by the model(1.5% in the data versus 1.35% in the model). The model generates higher volatility for gross output in consumption and equipment than in aggregate value added, and higher volatility in the equipment sector. The standard deviations of gross output in the consumption sector is 1.8% in the data and 2% as predicted by the model; while the standard deviation in the equipment sector is 4.8% in the data and 5.3% in the model. Finally, the model predicts autocorrelations close in magnitude to the ones observed in the data. The correlation predicted in the equipment is lower than in the consumption sector (as observed in the data).

8 Results

8.1 Aggregate and Sectoral Shocks in a flexible cost share economy

Table 10 presents the variance decomposition across shocks for aggregate value added, aggregate productivity, investment and consumption. Shocks to the equipment sector combined explain about 20% of the volatility of output in out baseline calibration. Roughly 60% of those movements are accounted by the volatility of the transitory component of the intermediate equipment sector,and the remaining to the persistent and transitory component of the innovations in the investment equipment sector. Neutral shocks explain the remaining of the volatility of GDP, with two thirds of it accounted by the transitory component of shocks, and the rest by persistent shocks.

s shown in the body of the paper, the aggregate production possibility frontier shifts with changes in the relative intensity with which inputs are used in production across different sectors, and the relevance of each sector for gross output. For our baseline calibration, 47% of the volatility of aggregate productivity is explained by shocks to the equipment sector; 27% of the fluctuations in aggregate productivity originate in persistent shocks. Neutral shocks explains 53% of the induced variation in TFP, with roughly 60% of it contributed by the transitory component. It is important to highlight that shocks to the intermediate production of equipment goods account for most of the volatility induced by shocks to the equipment sector. This model implies that changes in the productivity with which intermediate goods are produced have a stronger impact on aggregate productivity than shocks to the investment good production. Quantitatively the introduction of intermediate goods production is important in explaining changes in aggregate TFP.

The contribution of shocks to the volatility of aggregate investment is very similar to the one found for aggregate productivity. The volatility of aggregate consumption in the model is mostly explained by transitory shocks (87%), and most of it contributed by shocks to the investment equipment sector (67%). Shocks to the equipment intermediate sector barely affect aggregate consumption volatility.

8.2 Constant versus Flexible Cost Shares

In the previous section I highlighted the importance of modelling intermediate input in a the standard two sector economy, in particular for the impact of shocks in TFP and aggregate output. Now, I would like to show that allowing for a flexible cost share structure is also relevant in assessing the impact of neutral and sectoral shocks for aggregate volatility. To do so, I compare the calibrated flexible cost share economy with a comparable economy assuming a constant input output structure. I calibrate an economy where the elasticity of substitution is equal to one (Cobb Douglas technology) to generate the same input structure as the benchmark economy in steady state. The steady state input structure is important because the elasticities of inputs to changes in relative prices (in the flexible cost share economy) depend on the initial shares of inputs in production. I do this in two steps, first I only focus on generating the same input structure in steady state given the shock structure. As it turns out, the constant cost share economy, generates consistently lower volatility for the aggregates in the economy (i.e. value added, and gross output in each of the sectors), indicating that the amplification of shocks is weaker than in the flexible cost share economy. I hence recalibrate the shock structure to match the volatility and autocorrelation of GDP and gross output in the economy with flexible cost shares and compare results.

Table 11 reports the calibration of the model and compares it to the benchmark allocation. The shares of capital are identical across specifications, except in the intermediate equipment sector, where the share is computed to assure that the steady state is well defined. The elasticities of substitution across intermediate inputs are set to 1 ($\rho = 0$), so that the technology is Cobb Douglas in all inputs. The share of equipment (consumption) intermediates in the investment (consumption) sector is slightly higher than in the benchmark economy. As in the benchmark calibration, both models predict relatively well the input composition in the equipment sector, but do not account for all the disparity in intermediate input intake in the consumption sector.

Without adjustment of the cost structure the economy with constant cost shares generates much lower volatility of the series of GDP and gross output in the consumption and equipment sector. This is already a symptom of the differential impact of shocks in a flexible and constant cost share economy that I will describe through the predicted impulse response functions.

When I recalibrate the shocks to generate the moments of the flexible cost share economy I identify a standard deviation for the investment equipment sector shock twice as large as the one identified in the benchmark model. The standard deviation of the transitory component of the shock in the intermediate equipment sector is identified

50% as large as in the benchmark economy. Finally, while the transitory component of the neutral shock is as large as in the flexible cost share economy, the standard deviation of the persistent shock is identified almost twice as large and in the latter.

Variance Decomposition Table 13 displays the contribution to the variance of GDP, aggregate productivity, investment and aggregate consumption from neutral and sector specific shocks when we only match the industry structure across economies.

If we compare the variance decomposition of GDP, both the constant and flexible cost share economy predict similar contributions from temporary and persistent shocks. The benchmark economy predicts a slightly higher contribution of neutral shocks with value added volatility (2%) and a slightly lower contribution of temporary equipment sector shocks (3% in the investment sector and 1% in the equipment intermediate sector).

The contribution of neutral shocks to the variance in total factor productivity is lower in the constant cost share economy (5% in temporary shocks, and 2% for persistent shocks) and shocks to the intermediate equipment sector are predicted to explain relatively more of TFP variance. The variance decomposition of aggregate investment is very close across economies, and the variance of aggregate consumption is mostly explained by investment shocks as in the benchmark economy.

When the constant cost share economy is recalibrated to generate the volatility and autocorrelation of the aggregate series, the predictions of each of the models depart substantially. The economies that compare next generate the same steady state industry structure and the simulated series have the same statistical properties. Yet, the constant cost share economy predicts that 60% of the volatility of aggregate output is explained by shocks to the equipment sector while in the benchmark economy they explain 50%. The temporary (persistent) component of neutral shocks explains 19% (22%) of output volatility in the constant cost share economy and 35% (15%) in the benchmark one. Hence, the economy with a fixed input output structure predicts larger contribution for equipment sector shocks, but also if shifts the contribution of neutral shocks from temporary to persistent shocks.

This shift across type of shocks also occurs in the contributions to the volatility of aggregate TFP. More important, while investment equipment sector shocks are negligible in explaining TFP volatility in the benchmark economy, they are predicted to explain 27% of its volatility in the constant cost share economy. As described in the body of the paper the production possibility frontier of a multisector economy depends not only on the productivity of each sector, but also on the contribution of each of them to aggregate gross output. It is not surprising then that in a flexible cost share economy (where those contributions are responding to shifts in relative efficiency in production) the contribution of intermediate equipment sector shock to the volatility of TFP is larger than in the economy with a fixed input output structure (its transitory component contributes almost twice as much, and its persistent component 4 times more).

The comparison in the variance decomposition of aggregate investment is very close to the one described for TFP. In the case of consumption volatility, the role of transitory investment sector shocks is stronger in the constant cost share economy, reaching above 90% of consumption volatility.

Given that the predicted aggregate volatility in each economy differ, I recalibrate the constant cost share economy to generate volatility and persistence of the aggregate series closer to the ones predicted by the flexible cost share economy (See 15). Then, I compute a variance decomposition of the shocks and compare it to the baseline economy (as shown in 16).

The most salient feature in this comparison is that shocks to the equipment sector become substantially more relevant in explaining both aggregate output volatility and aggregate productivity volatility. While in the baseline case they only explain 20% of GDP volatility, they account for 45% of GDP volatility in the recalibrated economy. Also, while they explain roughly 45% of TFP volatility in the baseline economy, they are predicted to account for 60% of it in the recalibrated constant cost share economy. This shifts are substantial and depict the value in modelling carefully the input output structure, and in turn the variance covariance matrix of the underlying shocks.

Impulse Responses This section presents the predicted responses of key macroeconomic variables to shocks to productivity in alternative sectors. The focus in on a comparative analysis of responses in the flexible versus the constant cost share economy.

I study the behavior of aggregate output, TFP, aggregate consumption, gross output in the consumption sector, investment, and the relative price of new capital goods versus consumption goods. Figures 12 to 12 depict the responses of this variables to transitory neutral shocks, investment specific shocks, and shocks to the equipment intermediates sector. Figures 12 to 12 display the responses to shocks in the persistent component of productivity for the same three alternatives.

In general terms, on impact the economy with flexible cost shares reacts more to a given shock than the constant cost share economy does. Purely transitory shocks can have persistent effect on aggregate variables through the effect on equilibrium capital accumulation in each of the sectors. The persistent effect is stronger in the benchmark economy, except for aggregate TFP, which in both economies returns to its steady state level after one period.

Let me describe the response to each of the possible shocks one at the time. If the shock is neutral, a one standard deviation shock induces a three fold raise in aggregate output. In the constant cost share economy, the predicted increase in activity is 3/4 of the one predicted under flexible cost shares. As expected, aggregate consumption reacts less than aggregate value added, and investment and gross output in the consumption sector almost one to one with the increase in GDP. The relative price of intermediate inputs (equipment to consumption) drops on impact but convergences above steady state after that. The drop in the relative price in response to the neutral shock is slightly above 10% of its steady state level. Although the shock is neutral in nature, the disparity in input intensity across sectors generates changes in relative prices, and asymmetric responses of gross output in different sectors.

When the shock originates in the investment sector and it is persistent, one standard deviation shock (0.01) generates a reaction on GDP upon impact of 7% of the size of the shock. In the long run, GDP augments to 10% of the size of the shock in the investment sector. Gross final output falls upon impact to then overshoot its steady state level. This is explained mostly by the increase in the capital stock. The relative price of intermediate equipment goods raises 15% on impact and drops to slightly below 13% in the long run. In the constant cost share economy, the predicted change in relative prices is only 11% and converges to 9% in the long run. The disparity in the behavior of relative prices after impact is related to the more pronounced response of aggregate investment on impact for the flexible cost share economy. If the shock is purely transitory instead, gross value added is predicted to drop upon impact and raise above steady state levels after that. This is expected in response to the raise in investment upon impact.

Finally, if the shock originates in the intermediate equipment sector and is transitory, a one standard deviation shock (0.014) generates a positive reaction of GDP upon impact of about one third of the size of the shock. Gross output in the consumption sector increases half as much as the size of the shock in the flexible cost share economy, but slightly about a quarter of the size of the shock in the constant cost share economy. Investment increases 60% of the size of the shock and 40% in the constant cost share economy. The disparity in the behavior of gross output is mostly explained by the differences in the predictions for aggregate total factor productivity. While in the benchmark economy it is predicted to increase three times the size of the shock (recall that the Domar weights in the computation of TFP shift with the composition of gross output and value added in the economy) in the constant cost share economy, productivity raises two thirds of that.

In summary, while the dynamic predicted in either economy are comparable. Quantitatively the amplification of shocks in the flexible cost share economy is larger than in the constant cost share economy.

9 Conclusion

This paper studies the effect of fluctuations in the cost shares of intermediate inputs for the volatility of output. The model economy is calibrated to match the industry structure of the USA economy and the cyclical cost share behavior documented in the data. When tested against a comparable constant cost share economy, neutral shocks account for 8% more of aggregate output volatility. Responses of aggregate output and productivity to sectoral shocks are magnified when patterns of reallocation of factors are consistent with constant cost shares.

The disparities in cost share behavior across sectors may provide identifying restrictions for the nature of shocks to the economy. The results stems not only from the disparate contribution of sectors to value added, but also from degree of substitutability in inputs of production. Additional empirical analysis on the latter might be a promising avenue for further work. Furthermore, the disparities in the predictions of the dynamic of key aggregate variables may provide additional identification restrictions for the nature of shocks in the economy.

The paper illustrate the quantitative implications of disciplining the model economy to generate the pattern of reallocation observed in the data. It is still an open questions which are the mechanisms that generate those patterns. Are they consistent with factor specificity? do inventories play a role? do these patterns change when credit conditions change? Analysis of the input output structure dynamic for more disaggregated sector can shed light to some of these questions.

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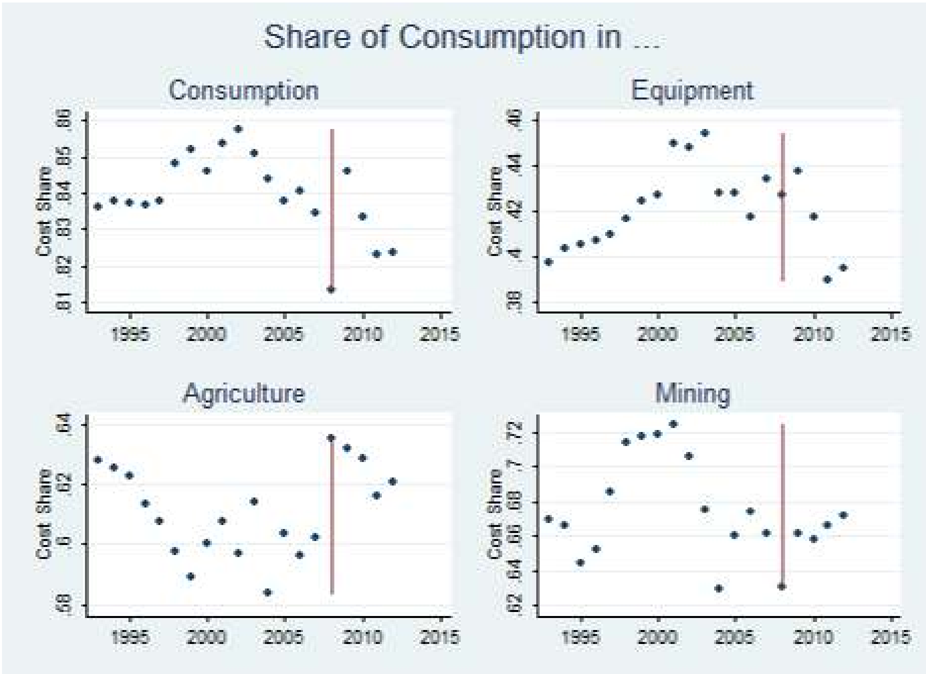


Figure 1: Consumption Cost Shares

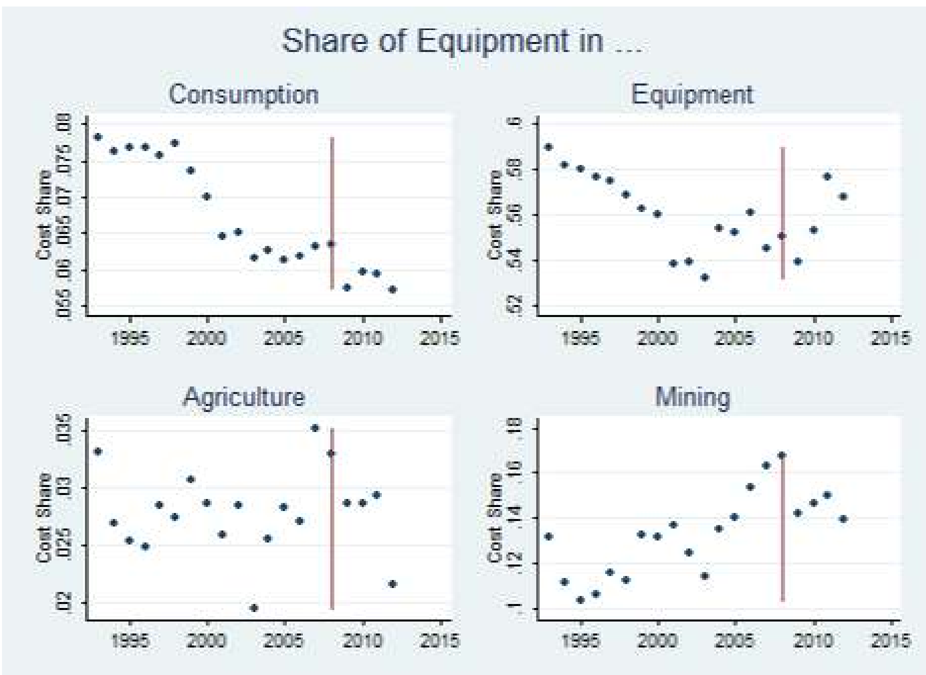


Figure 2: Equipment Cost Shares

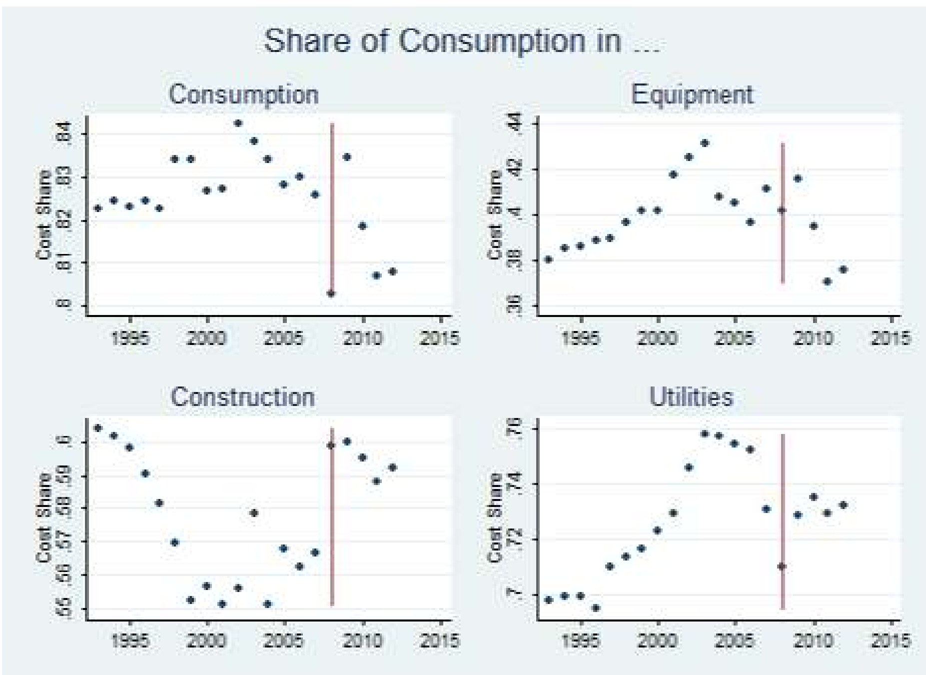


Figure 3: Consumption Cost Shares

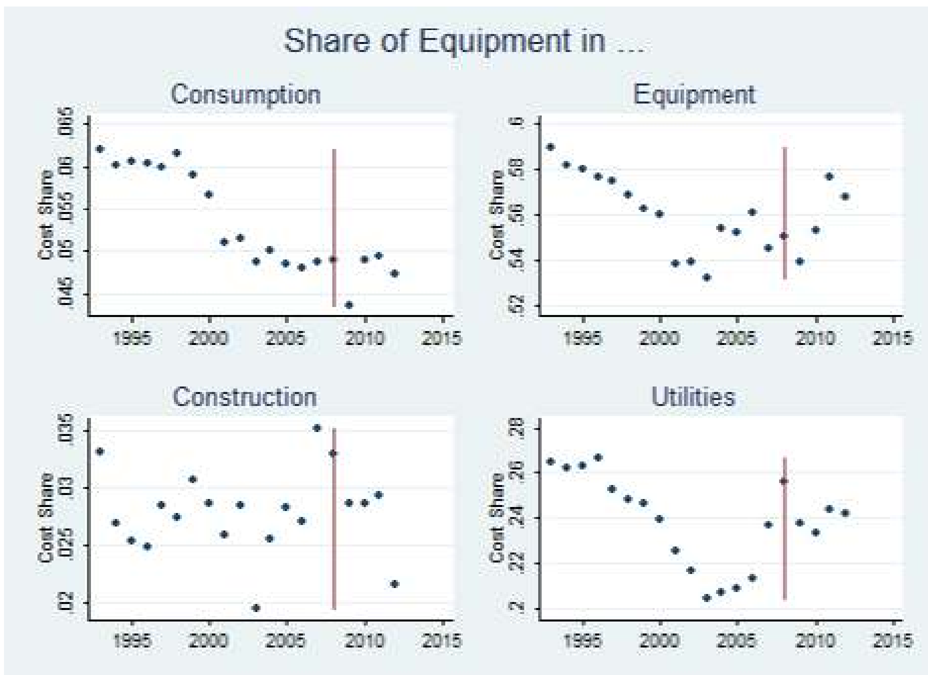


Figure 4: Equipment Cost Shares

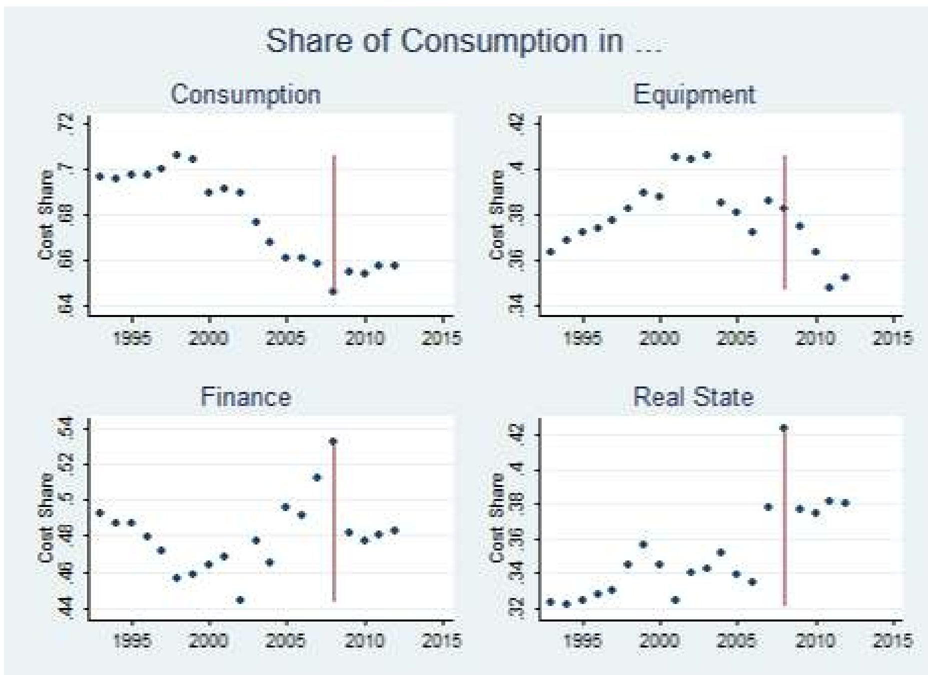


Figure 5: Consumption Cost Shares

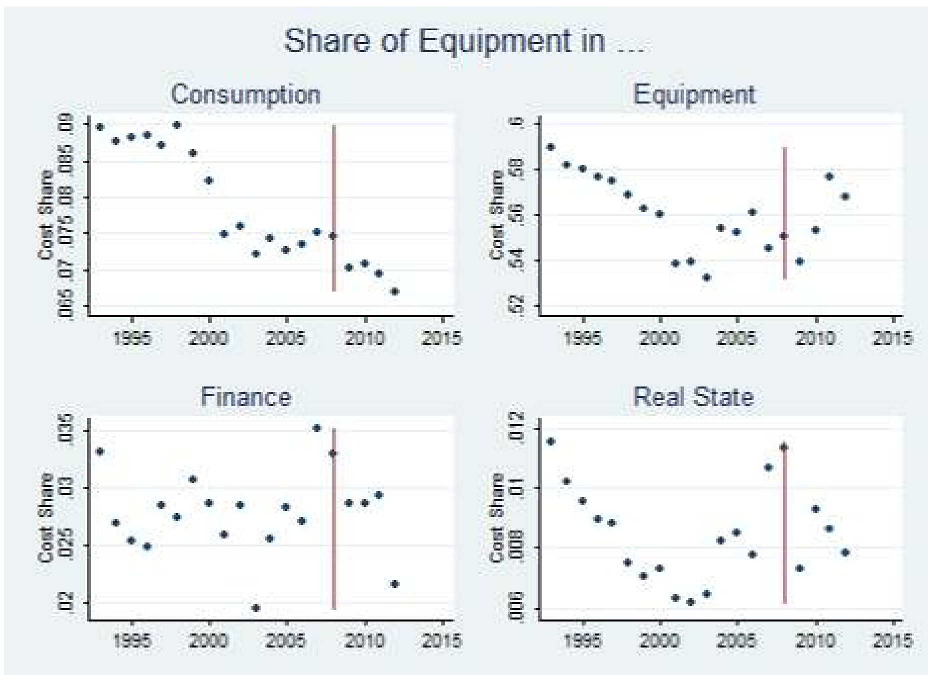


Figure 6: Equipment Cost Shares

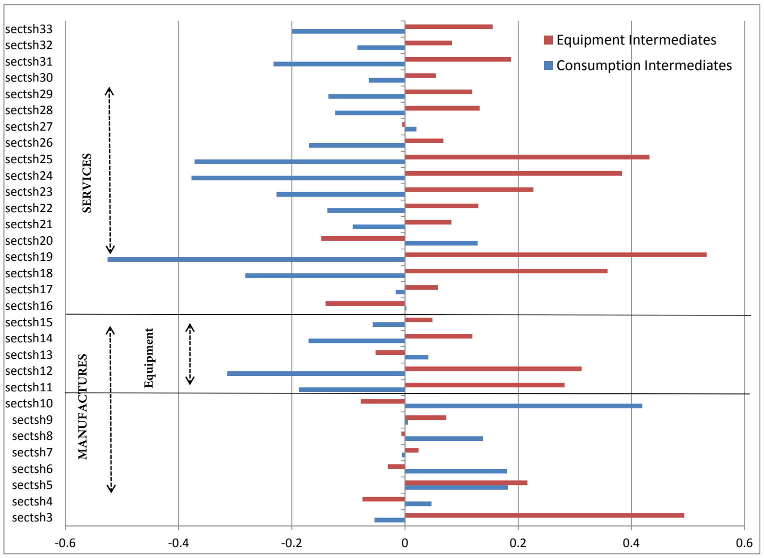


Figure 7: Correlation with Aggregate Output

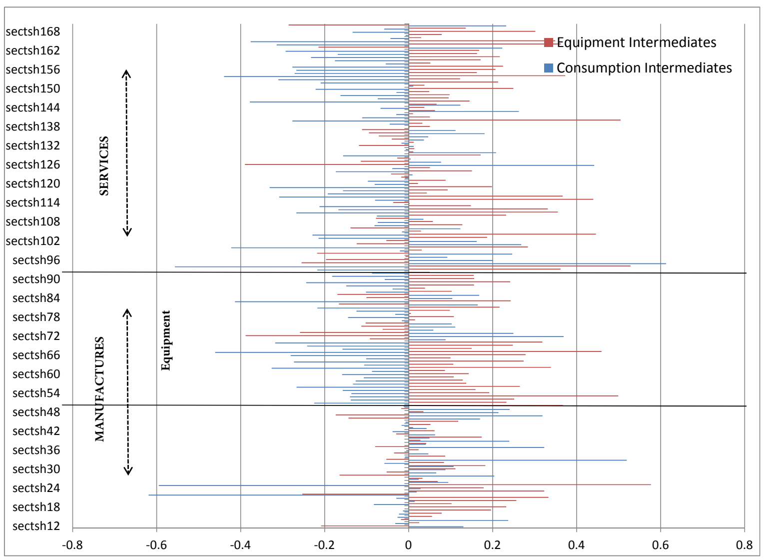


Figure 8: Correlation with Aggregate Output

10 Appendix

10.1 Balance Growth Path

Theorem 2 *A BGP where all intermediate goods are used in production exists iff technology is Cobb Douglas in capital and intermediates and either 1) the elasticity of substitution across intermediate goods, equals unity (Cobb-Douglas technology); or 2) there are no linkages through intermediate goods between the investment and consumption sector; 3) productivity growth in the consumption sector and intermediate equipment sector are proportional by a factor* $(g^x)^{\alpha_{y1}-\zeta}(g^y)^{\alpha_{my}-\alpha_{mx_2}}$ *,where* g^x *is the growth rate of output in the investment sector and* g^y *is the one in the consumption sector.*

1) and 2) are special cases of 3), hence I prove the latter first.

Proof. Suppose that productivity in the investment durable sector grows at rate γ^x

and productivity in the consumption and intermediate equipment sector grows at rate γ^y and γ^{x_2} respectively. Let g^j be the growth rate of output in sector $j = y, x, x_2$.

The feasibility restrictions in the economy imply

$$g^y = g^{M_{yy}} = g^{M_{yx}} = g^c$$

$$g^{x_2} = g^{M_{xx}} = g^{M_{xy}}$$

$$g^x = g^{i_j} = g^{k_j} \text{ for any } j = y, x, x_2$$

Given production technologies, for intermediate goods from the consumption and equipment sector to be used in production along the BGP, the growth rate of output in the consumption and intermediate equipment should equalize. Such feature stems from the optimality conditions of the firms in intermediate inputs.

$$\alpha_{my}\alpha_{y2}\left(\frac{Y}{M_{yy}}\right)\frac{1}{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}=1 \tag{4}$$

Unless the growth rates of input intake from the equipment and consumption sector are the same, the optimality condition would not be satisfied along the BGP.

$$g^{x_2}=g^y$$

From the production technology in the consumption sector and intermediate equipment sector we obtain

$$g^y=\gamma^y\left(g^x\right)^{\alpha_{y1}}\left(g^y\right)^{\alpha_{my}}$$

$$g^{x_2}=\gamma^{x_2}\left(g^x\right)^{\varsigma}\left(g^y\right)^{\alpha_{mx_2}}$$

Hence,

$$\gamma^{x_2}=\gamma^y\left(g^x\right)^{\alpha_{y1}-\zeta}\left(g^y\right)^{\alpha_{my}-\alpha_{mx_2}}$$

which depends on the relative capital intensity of the consumption sector and intermediate equipment sector.

Finally, from the investment sector technology, we have

$$g^x=\gamma^x\left(g^x\right)^{\alpha_{x1}}\left(g^y\right)^{1-\alpha_{x1}}$$

Combining this equation with the one describing growth rates in the consumption sector, we obtain the BGP of the economy, i.e.

$$g^x=\left(\gamma^x\right)^{\psi_{x1}}\left(\gamma^y\right)^{\psi_{x2}}$$

$$g^y=\left(\gamma^x\right)^{\psi_{y1}}\left(\gamma^y\right)^{\psi_{y2}}$$

$\psi_{x1} = \frac{1-\alpha_{my}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{x2} = \frac{a_{mx}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$. Also, $\psi_{y1} = \frac{a_{y_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$ and $\psi_{y2} = \frac{1-a_{x_1}}{(1-\alpha_{x1})(1-\alpha_{my})-\alpha_{mx}a_{y1}}$

To show number 1), note that the problem that was pointed out in 4 is not present anymore, for $\rho = 0$. Hence, the condition $g^{x_2} = g^y$ need not hold. The algebra gets more cumbersome but it is possible to show that the BGP will solve

$$g^x = \gamma^x (g^x)^{\alpha_{x1}} \left((g^{x_2})^{\alpha_{x2}} (g^y)^{1-\alpha_{x2}}\right)^{\alpha_{mx}}$$

$$g^y = \gamma^y (g^x)^{\alpha_{y1}} \left((g^{x_2})^{1-\alpha_{y2}} (g^y)^{\alpha_{y2}}\right)^{\alpha_{my}}$$

$$g^{x_2} = \gamma^{x_2} (g^x)^{\varsigma} (g^{x_2})^{\alpha_{mx_2}}$$

Number 2) is analogous to number 3) but now the system of equations to be solved is

$$g^x = \gamma^x (g^x)^{\alpha_{x1}} ((g^{x_2})^{\alpha_{x2}})^{\alpha_{mx}}$$

$$g^y = \gamma^y (g^x)^{\alpha_{y1}} ((g^y)^{\alpha_{y2}})^{\alpha_{my}}$$

$$g^{x_2} = \gamma^{x_2} (g^x)^{\varsigma} (g^{x_2})^{\alpha_{mx_2}}$$

■

11 Appendix (C)

11.1 Optimality and Steady State

Feasibility dictates

$$k_{x1}^{'}(1+g^{x})-k_{x1}(1-\delta)=i_{x1} \qquad (\kappa_x)$$

$$k_{x2}^{'}(1+g^{x})-k_{x2}(1-\delta)=i_{x2}$$

$$k_{y}^{'}(1+g^{y})-k_{y}(1-\delta)=i_{y} \qquad (\kappa_y)$$

$$i_y+i_{x1}+i_{x2}=X_1 \qquad (\lambda_x)$$

$$M_{xx}+M_{xy}=X_2 \qquad (\lambda_{x2})$$

$$C+M_{yy}+M_{yx}=Y \qquad (\lambda_y)$$

$$M_{yx}=M_{yx1}+M_{yx2}$$

The corresponding optimality conditions are

$$\lambda_x(1+g^x)=\beta\lambda_x'\left[\alpha_{x1}\left(\frac{X_1'}{k_{x_1}'}\right)+(1-\delta)\right]$$

$$\lambda_x(1+g^{x_2})=\beta\lambda_x'\left[\frac{\lambda_{x2}'}{\lambda_x'}\zeta_x\left(\frac{X_2'}{k_{x_2}'}\right)+(1-\delta)\right]$$

$$\lambda_x(1+g^{y})=\beta\lambda_x'\left[\frac{\lambda_{y}'}{\lambda_x'}\alpha_{y1}\left(\frac{Y'}{k_{y}'}\right)+(1-\delta)\right]$$

$$\left(1-\alpha_{y1}\right)\alpha_{y2}\left(\frac{Y}{M_{yy}}\right)\frac{\left(M_{yy}\right)^{\rho_y}}{\alpha_{y2}\left(M_{yy}\right)^{\rho_y}+\left(1-\alpha_{y2}\right)\left(M_{xy}\right)^{\rho_y}}=1 \qquad (M_{yy})$$

$$\lambda_y\left(1-\alpha_{y1}\right)\left(1-\alpha_{y2}\right)\left(\frac{Y}{M_{xy}}\right)\frac{\left(M_{xy}\right)^{\rho_y}}{\alpha_{y2}\left(M_{yy}\right)^{\rho_y}+\left(1-\alpha_{y2}\right)\left(M_{xy}\right)^{\rho_y}}=\lambda_{x2} \qquad (M_{xy})$$

$$\lambda_x \left(1-\alpha_{x1}\right)\alpha_{x2}\left(\frac{X_1}{M_{xx}}\right)\frac{\left(M_{xx}\right)^{\rho_x}}{\alpha_{x2}\left(M_{xx}\right)^{\rho_x}+\left(1-\alpha_{x2}\right)\left(M_{yx1}\right)^{\rho_x}}=\lambda_{x2} \qquad (M_{xx})$$

$$\lambda_x \left(1-\alpha_{x1}\right)\left(1-\alpha_{x2}\right)\left(\frac{X_1}{M_{yx1}}\right)\frac{\left(M_{yx1}\right)^{\rho_x}}{\alpha_{x2}\left(M_{xx}\right)^{\rho_x}+\left(1-\alpha_{x2}\right)\left(M_{yx1}\right)^{\rho_x}}=\lambda_y \qquad (M_{yx})$$

$$\lambda_{x2}(1-\zeta_x)\left(\frac{X_2}{M_{yx2}}\right)=\lambda_y \qquad (M_{yx2})$$

This is a standard convex economy. Hence, the equilibrium exits and its unique. Also the welfare theorems hold.

11.2 Steady State

From the production function, we obtain

$$\frac{X_1}{M_{xx}}=\left(\frac{k_{x1}}{M_{xx}}\right)^{\alpha_{x1}}\left(\alpha_{x2}+\left(1-\alpha_{x2}\right)\left(\frac{M_{yx1}}{M_{xx}}\right)^{\rho_x}\right)^{\frac{1-\alpha_{x1}}{\rho_x}}$$

Using the optimality condition in intermediate goods and capital we can rewrite the equation as

$$\frac{\lambda_{x2}}{\lambda_x}\frac{1}{\left(1-\alpha_{x1}\right)\alpha_{x2}}=\left(\frac{\alpha_{x1}}{\left(1-\alpha_{x1}\right)\alpha_{x2}}\frac{\beta}{1+g^x-\beta(1-\delta)}\frac{\lambda_{x2}}{\lambda_x}\right)^{\alpha_{x1}}\left[\alpha_{x2}+\left(1-\alpha_{x2}\right)\left(\frac{1-\alpha_{x2}}{\alpha_{x2}}\frac{\lambda_x}{\lambda_y}\right)^{\frac{\rho_x}{1-\rho_x}}\right]^{(1-\alpha_{x1}} \qquad (5)$$

which defines the equilibrium relative prices of investment goods versus consumption goods.

Using the production function in the final good sector we can solve for $\frac{\lambda_{x2}}{\lambda_y}$ as

$$\frac{Y}{M_{yy}}=\left(\frac{k_y}{M_{yy}}\right)^{\alpha_{y1}}\left(\alpha_{y2}+\left(1-\alpha_{y2}\right)\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}\right)^{\frac{1-\alpha_{y1}}{\rho_y}}$$

Following the same procedure as before, we can express this equation as a function of the relative price of investment and final goods.

$$\frac{1}{(1-\alpha_{y1})\,\alpha_{y2}}=\left(\frac{\alpha_{y1}}{(1-\alpha_{y1})\,\alpha_{y2}}\frac{\beta}{1+g^{y}-\beta(1-\delta)}\frac{\lambda_y}{\lambda_x}\right)^{\alpha_{y1}}\left[\alpha_{y2}+(1-\alpha_{y2})\left(\frac{1-\alpha_{y2}}{\alpha_{y2}}\frac{\lambda_y}{\lambda_{x2}}\right)^{\frac{\rho_y}{1-\rho_y}}\right]^{\left(1-\alpha_{y1}\right)\frac{1-\rho_y}{\rho_y}} \tag{6}$$

Finally, the production technology in the third sector dictates

$$\frac{X_2}{k_{x2}}=\left(\frac{M_{yx2}}{k_{x2}}\right)^{(1-\varsigma_x)}$$

$$\left(\frac{1+g^{y}-\beta(1-\delta)}{\beta}\frac{\lambda_x}{\lambda_{x2}}\frac{1}{\varsigma_x}\right)=\left(\frac{1+g^{y}-\beta(1-\delta)}{\beta}\frac{\lambda_x}{\lambda_y}\frac{1-\varsigma_x}{\varsigma_x}\right)^{(1-\varsigma_x)} \tag{7}$$

Hence, equations (5), (6) , (7) define a system of three equations and three unknowns. Given the calibrated parameters I impose conditions on the share of value added in the third sector so that the system is exactly determined.

From the feasibility condition in intermediate goods of the investment sector we obtain

$$\frac{M_{xx}}{k_{x2}}+\frac{M_{xy}}{k_{x2}}=\frac{X_2}{k_{x2}}$$

$$\frac{\lambda_x}{\lambda_{x2}}\frac{(1-\alpha_{x1})\,\alpha_{x2}}{\alpha_{x2}+(1-\alpha_{x2})\left(\frac{1-\alpha_{x2}}{\alpha_{x2}}\frac{\lambda_{x2}}{\lambda_y}\right)^{\frac{\rho_x}{1-\rho_x}}}\frac{X_1}{X_2}+\frac{\lambda_y}{\lambda_{x2}}\frac{(1-\alpha_{y1})\,(1-\alpha_{y2})}{\alpha_{y2}\left(\frac{\alpha_{y2}}{1-\alpha_{y2}}\frac{\lambda_{x2}}{\lambda_y}\right)^{\frac{\rho_y}{1-\rho_y}}+(1-\alpha_{y2})}\frac{Y}{X_2}=1$$

which determines the ratio of gross output in the production of equipment, as well as the ratio of consumption good production to intermediate investment goods, as a function of parameters and equilibrium prices.

If we now turn to the feasibility condition in the final production investment sector, we have

$$\delta(1+\frac{k_y}{k_x}+\frac{k_{x2}}{k_x})=\frac{X_1}{k_x}$$

$$\delta(1+\frac{\lambda_y}{\lambda_x}\frac{Y}{X_2}\frac{\alpha_{y1}}{\alpha_{x1}}\frac{X_2}{X_1}+\left(\frac{\lambda_{x2}}{\lambda_x}\right)\frac{\zeta_x}{\alpha_{x1}}\frac{X_2}{X_1})=\frac{1+g^x-\beta(1-\delta)}{\beta}\frac{1}{\alpha_{x1}}$$

If we put both feasibility conditions together we obtain a system of two equations in two unknowns, i.e. the ratios of gross output across sectors.

To pin down the levels of the variables use the feasibility constraint in the consumption good sector.

$$\frac{U^{-1}(\lambda_y^*)}{M_{yy}}+1+\frac{M_{yx}}{M_{yy}}=\frac{Y}{M_{yy}}$$

where $\frac{Y}{M_{yy}}=\frac{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}{(1-\alpha_{y1})\alpha_{y2}}$. As we have shown before, $\frac{M_{xy}}{M_{yy}}$ is a function of the prices in the economy. In other words, M_{yy} solves

$$\begin{aligned}&\frac{U^{-1\prime}(\lambda_y^*)}{M_{yy}}+1+\frac{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}{(1-\alpha_{y1})\,\alpha_{y2}}[\frac{X_1}{Y}\frac{\lambda_x}{\lambda_y}\frac{(1-\alpha_{x1})\,(1-\alpha_{x2})}{\alpha_{x2}\left(\frac{M_{xx}}{M_{yx}}\right)^{\rho_x}+(1-\alpha_{x2})}\\ &+\left(\frac{\lambda_{x2}}{\lambda_y}\right)\frac{(1-\varsigma_x)}{(1-\alpha_{y1})\,\alpha_{y2}}\frac{X_2}{Y}]\\ =\;&\frac{\alpha_{y2}+(1-\alpha_{y2})\left(\frac{M_{xy}}{M_{yy}}\right)^{\rho_y}}{(1-\alpha_{y1})\,\alpha_{y2}}\end{aligned}$$

where $\frac{M_{xy}}{M_{yy}}=\left(\frac{\lambda_y}{\lambda_{x2}}\frac{1-\alpha_{y2}}{\alpha_{y2}}\right)^{\frac{1}{1-\rho_y}}$ and $\frac{M_{xx}}{M_{yx}}=\left(\frac{\alpha_{x2}}{1-\alpha_{x2}}\frac{\lambda_y}{\lambda_{x2}}\right)^{\frac{1}{1-\rho_x}}$.

Once M_{yy}^* is determined M_{yx}^* is too, as well as Y^*, k_y^*, M_{xy}^* from the optimality conditions. k_x^* is determined using $\frac{k_y}{k_x}^*$ and then K^* can be computed. X^* is solved by the equilibrium ratio $\frac{X}{Y}^*$ and Y^*.

12 Appendix (D)

Parameter	Definition	Model
α_{x1}	Capital Share, Equipment	0.182
α_{y1}	Capital Share, Consumption	0.210
ς	Capital Share, Equipment Intermediates	0.901
θ_g	Consumption Shock Persistence	0.2
θ_{x_2}	Equipment Intermed. Shock Persistence	0.25
θ_{x_1}	Investment Shock Persistence	0.25
g^x	Gross Output Growth Rate, Equipment	3.15%
β	Discount Factor	0.98
δ	Capital Depreciation	0.05

Table 7: Parameters calibrated outside the model

Parameter	Definition	Value
ρ_x	Elasticity of Substitution M^{xx_1}, M^{yx_1}	-0.12
α_{x2}	Share Equipment Intermediates, Equipment	0.579
ρ_y	Elasticity of Substitution M^{yy}, M^{xy}	-1.36
α_{y2}	Share Consumption Intermediates, Consumption	0.658
	Volatility Transitory Shocks	
σ_ε^g	Neutral	0.0023
$\sigma_\varepsilon^{x_1}$	Investment	0.01
$\sigma_\varepsilon^{x_2}$	Intermediate Equipment	0.014
$Corr\left(\varepsilon^{x2}, \varepsilon^{x_1}\right)$	Covariance Equipment Sector	-0.728
	Volatility Persistent Shocks	
σ_η^g	Neutral	0.0016
σ_η^{x1}	Investment	0.0033
σ_η^{x2}	Intermediate Equipment	0.0001

Table 8: Jointly calibrated Parameters

Moment	Model	Data
Cost Shares		
Consumption Goods in Consumption	0.81	0.93
Consumption Goods in Equipment	0.52	0.57
Correlation, GDP Cycle and Cost Shares		
Consumption Goods in Consumption	-0.27	-0.13
Equipment Goods in Consumption	0.10	0.13
Equipment Goods in Equipment	0.22	0.26
Consumption Goods in Equipment	-0.22	-0.26
Standard Deviation		
GDP	0.013	0.015
Gross Output, Consumption	0.02	0.018
Gross Output, Equipment	0.053	0.0476
Autocorrelation (1)		
GDP	0.36	0.38
Gross Output, Consumption	0.39	0.36
Gross Output, Equipment	0.36	0.29

Table 9: Moments

	Transitory, ε			Persistent, η		
	A^g	A^{x_1}	A^{x_2}	A^g	A^{x_1}	A^{x_2}
Gross Domestic Product	52.8	8.12	12.1	26.8	0.08	0.00
Total Factor Productivity	35.4	9.22	28.58	18.39	8.67	0.00
Investment	34	10.5	28.8	17.6	9.1	0.00
Consumption	14	66.8	1.73	6.99	10.5	0.00

Table 10: Variance Decomposition, Baseline

PARAMETER	DEFINITION	CES	COBB-DOUGLAS
ρ_x	Elasticity of substitution, Equipment	-0.12	0
α_{x2}	Share of equipment intermediates, Equipment	0.579	0.49
ρ_y	Elasticity of substitution, Consumption	-1.36	0
α_{y2}	Share of consumption intermediates, Consumption	0.658	0.81
α_{x1}	Share of capital, Equipment	0.182	0.182
ς	Share of capital, Equipment intermediates	0.901	0.973
α_{y1}	Share of capital, Consumption	0.210	0.210

Table 11: Parametrization, Baseline CES vs. Cobb Douglas technology

MOMENT	CES	CD	DATA
Cost Shares			
Consumption Goods in Consumption	0.81	0.81	0.93
Consumption Goods in Equipment	0.52	0.52	0.57
Correlation, GDP Cycle and Cost Shares			
Consumption Goods in Consumption	-0.27	.	-0.13
Equipment Goods in Consumption	0.10	.	0.13
Equipment Goods in Equipment	0.22	.	0.26
Consumption Goods in Equipment	-0.22	.	-0.26
Standard Deviation			
GDP	0.013	0.01	0.015
Gross Output, Consumption	0.02	0.015	0.018
Gross Output, Equipment	0.053	0.041	0.0476
Autocorrelation (1)			
GDP	0.36	0.26	0.38
Gross Output, Consumption	0.39	0.41	0.36
Gross Output, Equipment	0.36	0.37	0.29

Table 12: Moments

	Transitory, ε			Persistent, η		
GDP	A^g	A^{x_1}	A^{x_2}	A^g	A^{x_1}	A^{x_2}
CES	52.8	8.12	12.1	26.8	0.08	0.00
Cobb-Douglas	47.7	12.09	15.6	24.4	0.06	0.00
TPF						
CES	35.4	9.22	28.58	18.4	8.67	0.00
Cobb-Douglas	39	20.26	12.57	20.17	7.8	0.00
Investment						
CES	34	10.5	28.8	17.6	9.1	0.00
Cobb-Douglas	34.5	12.52	25.9	17.9	9.1	0.00
Consumption						
CES	14	66.8	1.73	6.99	10.5	0.00
Cobb-Douglas	8.06	78.1	0.12	4.03	9.71	0.00

Table 13: Variance Decomposition

Parameter	Definition	CES	CD
	Volatility Transitory Shocks		
σ_{ε}^{g}	Neutral	0.0023	0.0036
$\sigma_{\varepsilon}^{x_1}$	Investment	0.01	0.028
$\sigma_{\varepsilon}^{x_2}$	Intermediate Equipment	0.014	0.037
$Corr\left(\varepsilon^{x2}, \varepsilon^{x_1}\right)$	Covariance Equipment Sector	-0.72	-0.72
	Volatility Persistent Shocks		
σ_{η}^{g}	Neutral	0.0016	0.0036
σ_{η}^{x1}	Investment	0.0033	0.0068
σ_{η}^{x2}	Intermediate Equipment	0.0001	0.0009

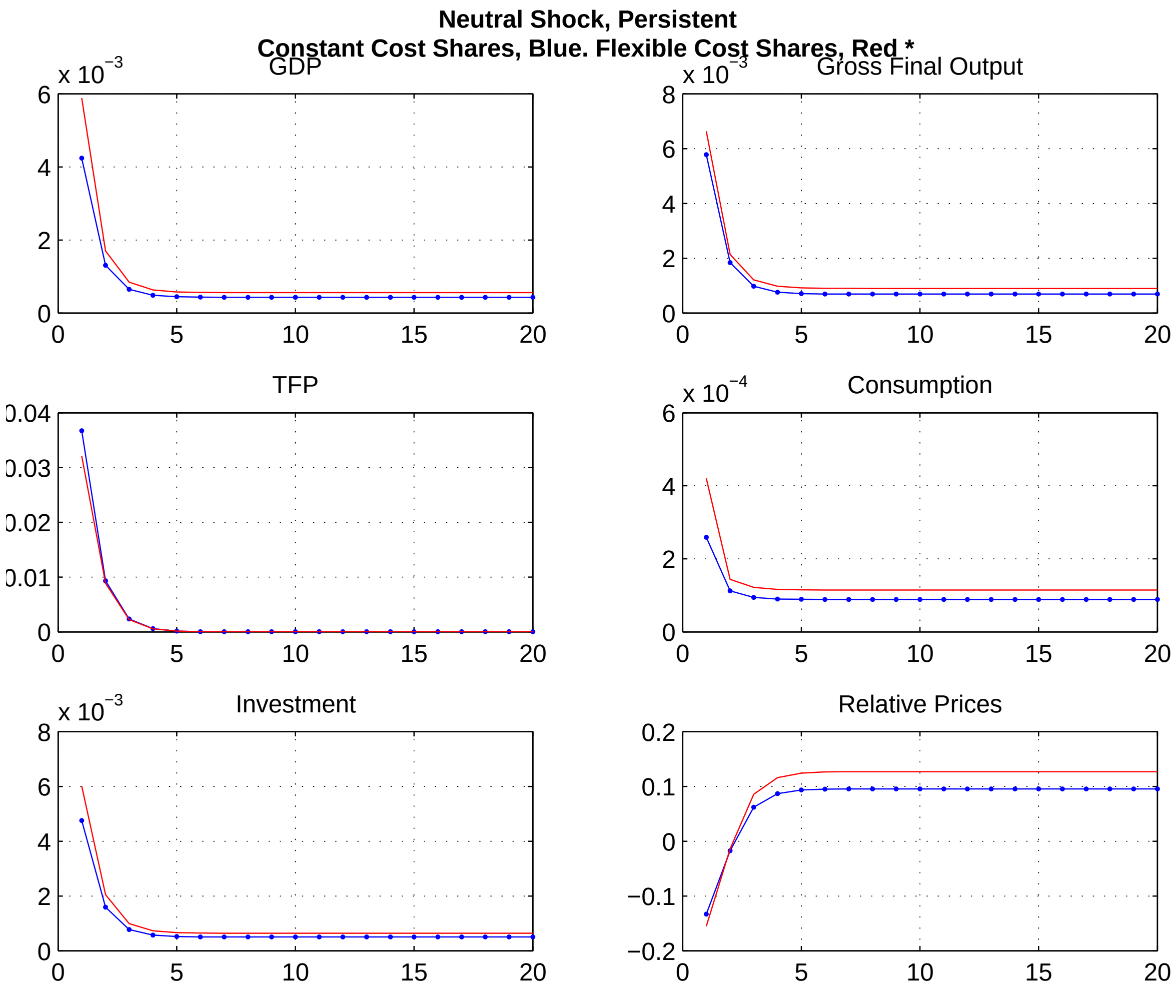
Table 14: Calibrated Shocks, Constant Cost Share Economy

Moment	CES	CD	CD Recalibrated
Cost Shares			
Consumption Goods in Consumption	0.81	0.81	0.81
Consumption Goods in Equipment	0.52	0.52	0.52
Correlation, GDP Cycle and Cost Shares			
Consumption Goods in Consumption	-0.27	.	.
Equipment Goods in Consumption	0.10	.	.
Equipment Goods in Equipment	0.22	.	.
Consumption Goods in Equipment	-0.22	.	.
Standard Deviation			
GDP	0.013	0.01	0.023
Gross Output, Consumption	0.02	0.015	0.033
Gross Output, Equipment	0.053	0.041	0.032
Autocorrelation (1)			
GDP	0.36	0.26	0.37
Gross Output, Consumption	0.39	0.41	0.40
Gross Output, Equipment	0.36	0.37	0.37

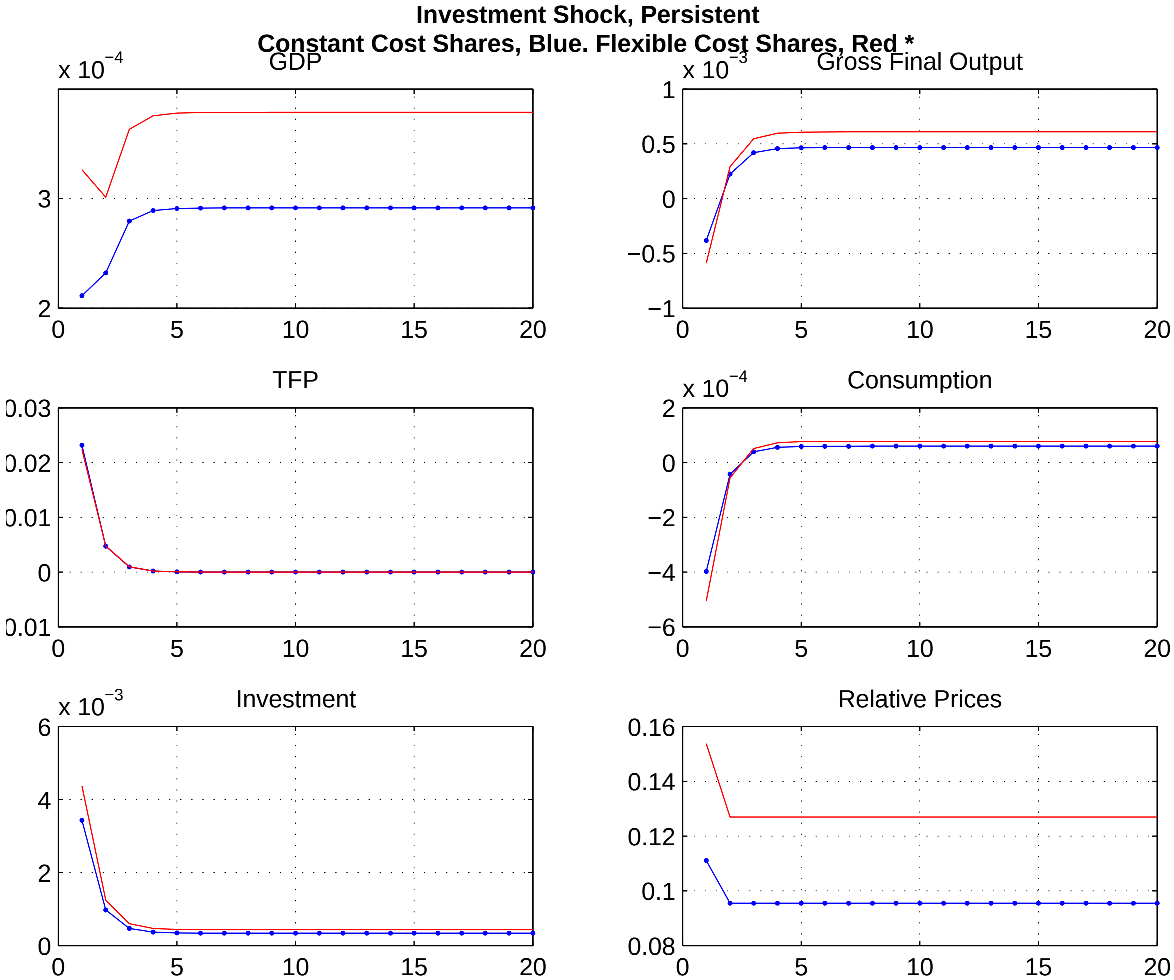
Table 15: Moments

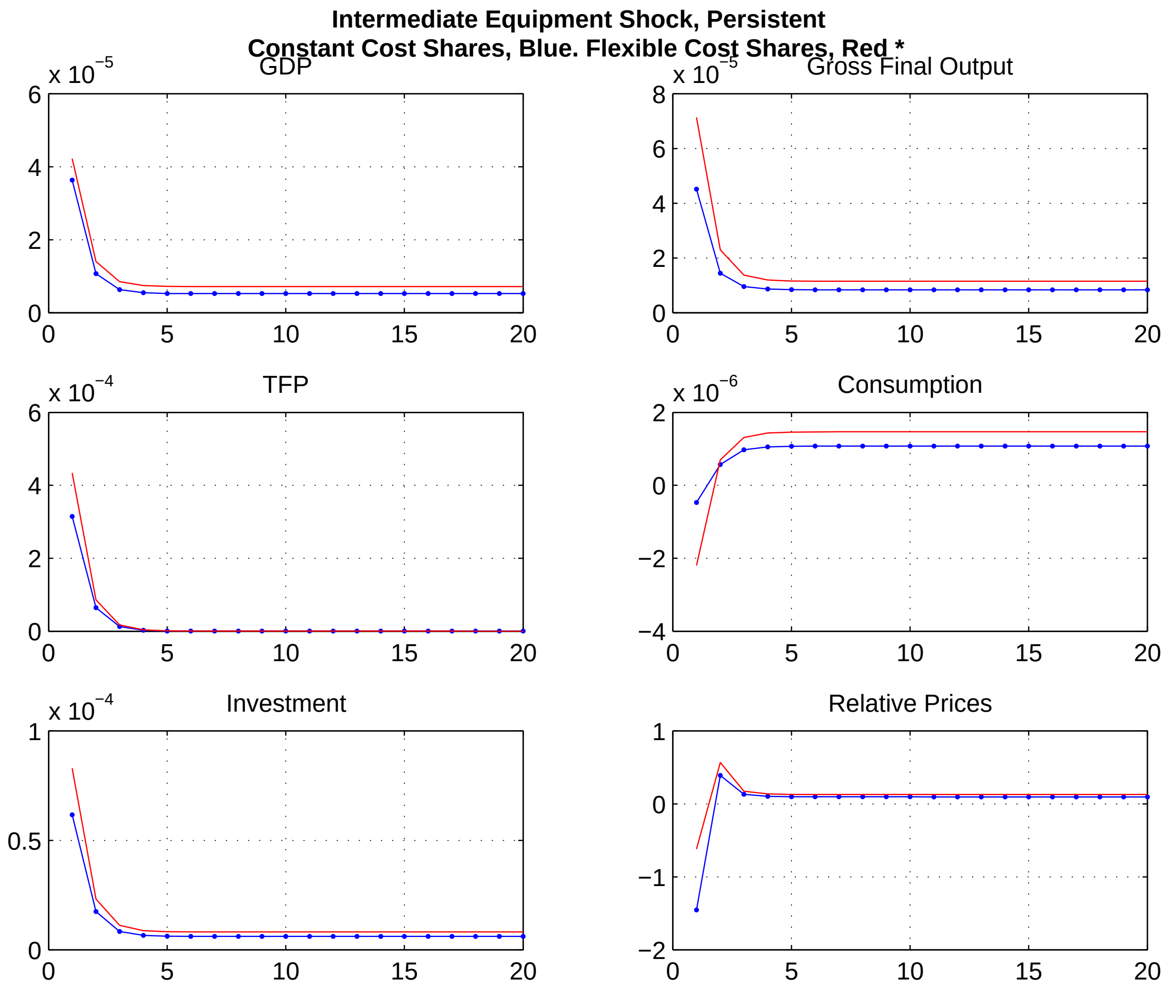
	Transitory, ε			Persistent, η		
GDP	A^g	A^{x_1}	A^{x_2}	A^g	A^{x_1}	A^{x_2}
`CES`	52.8	8.12	12.1	26.8	0.08	0.00
`Cobb-Douglas*`	26.7	19.4	25.3	28.2	0.06	0.03
TPF						
`CES`	35.4	9.22	28.58	18.4	8.67	0.00
`Cobb-Douglas*`	19.4	34.6	18.5	20.7	6.73	0.02
Investment						
`CES`	34	10.5	28.8	17.6	9.1	0.00
`Cobb-Douglas*`	16.6	21.8	36.3	17.7	7.5	0.05
Consumption						
`CES`	14	66.8	1.73	6.99	10.5	0.00
`Cobb-Douglas*`	2.8	88.2	0.12	2.93	5.92	0.00

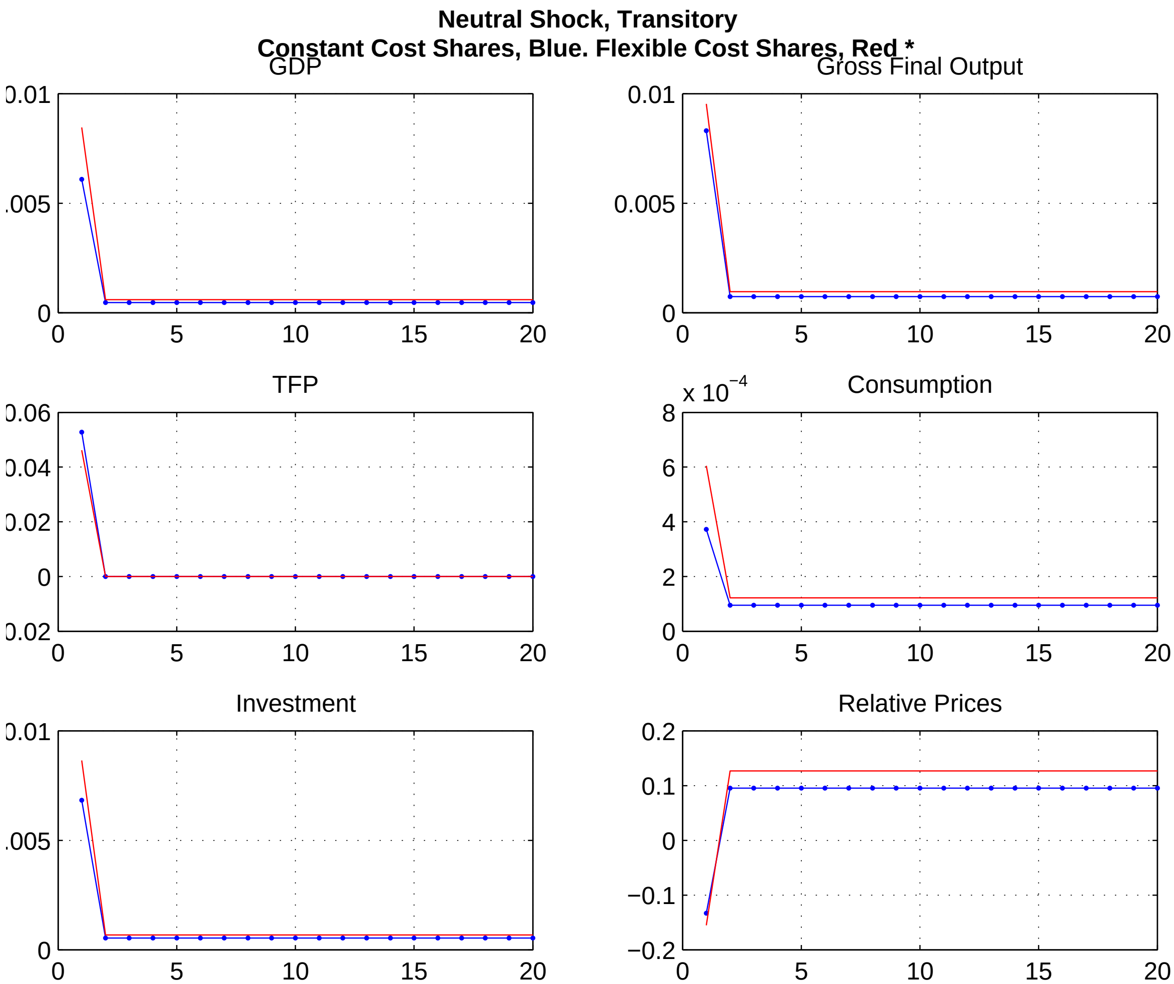
Table 16: Variance Decomposition: Cobb Douglas Economy, recalibrated

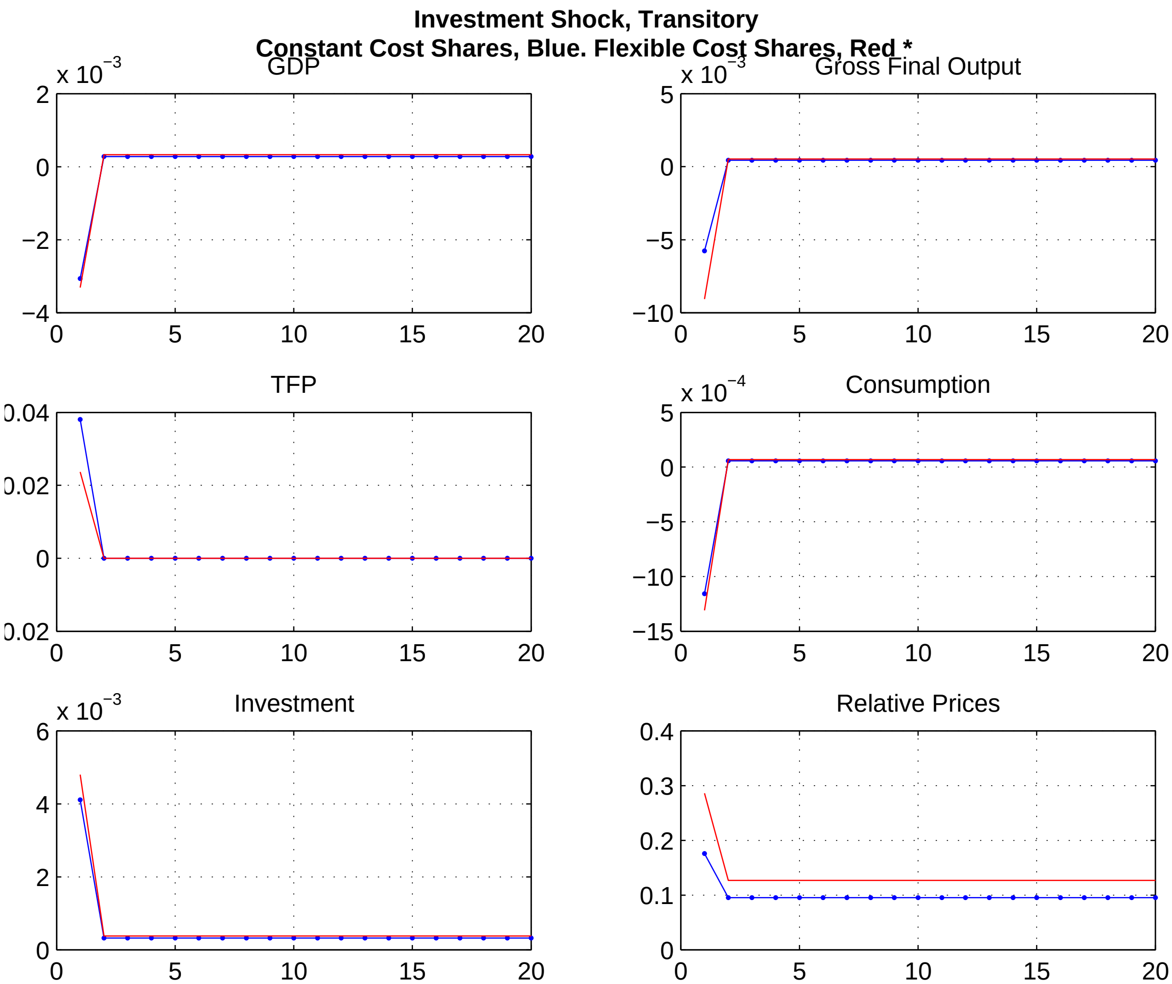


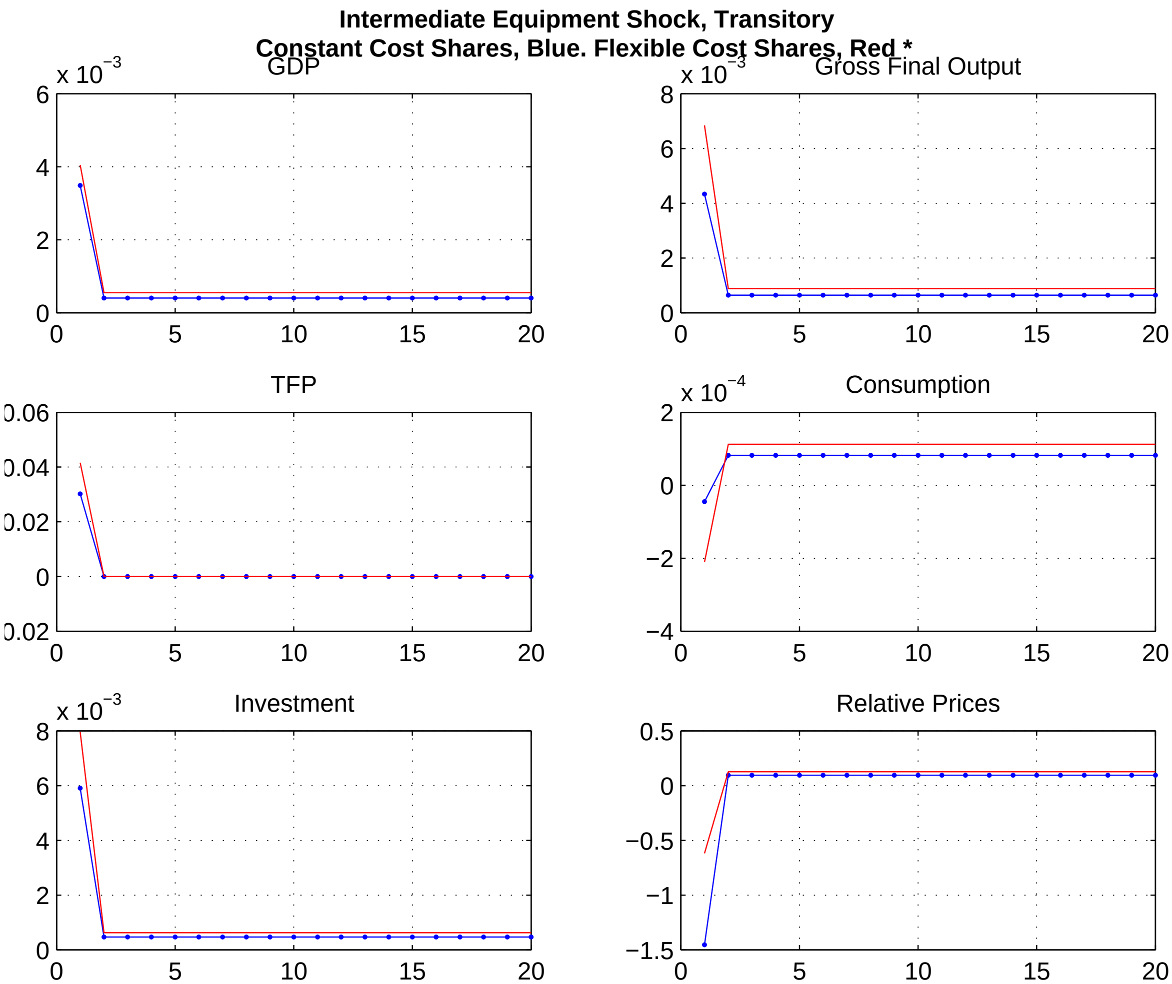
Neutral Shock, Persistent
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^−3
Gross Final Output
x 10^−3
TFP
Consumption
x 10^−4
Investment
x 10^−3
Relative Prices


Investment Shock, Persistent
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^−4
Gross Final Output
x 10^−3
TFP
Consumption
x 10^−4
Investment
x 10^−3
Relative Prices


Intermediate Equipment Shock, Persistent
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^−5
Gross Final Output
x 10^−5
TFP
x 10^−4
Consumption
x 10^−6
Investment
x 10^−4
Relative Prices
0
5
10
15
20


Neutral Shock, Transitory
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
Gross Final Output
TFP
Consumption
x 10^−4
Investment
Relative Prices


Investment Shock, Transitory
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^−3
Gross Final Output
x 10^−3
TFP
Consumption
x 10^−4
Investment
x 10^−3
Relative Prices



Intermediate Equipment Shock, Transitory
Constant Cost Shares, Blue. Flexible Cost Shares, Red *
GDP
x 10^-3
Gross Final Output
x 10^-3
TFP
Consumption
x 10^-4
Investment
x 10^-3
Relative Prices

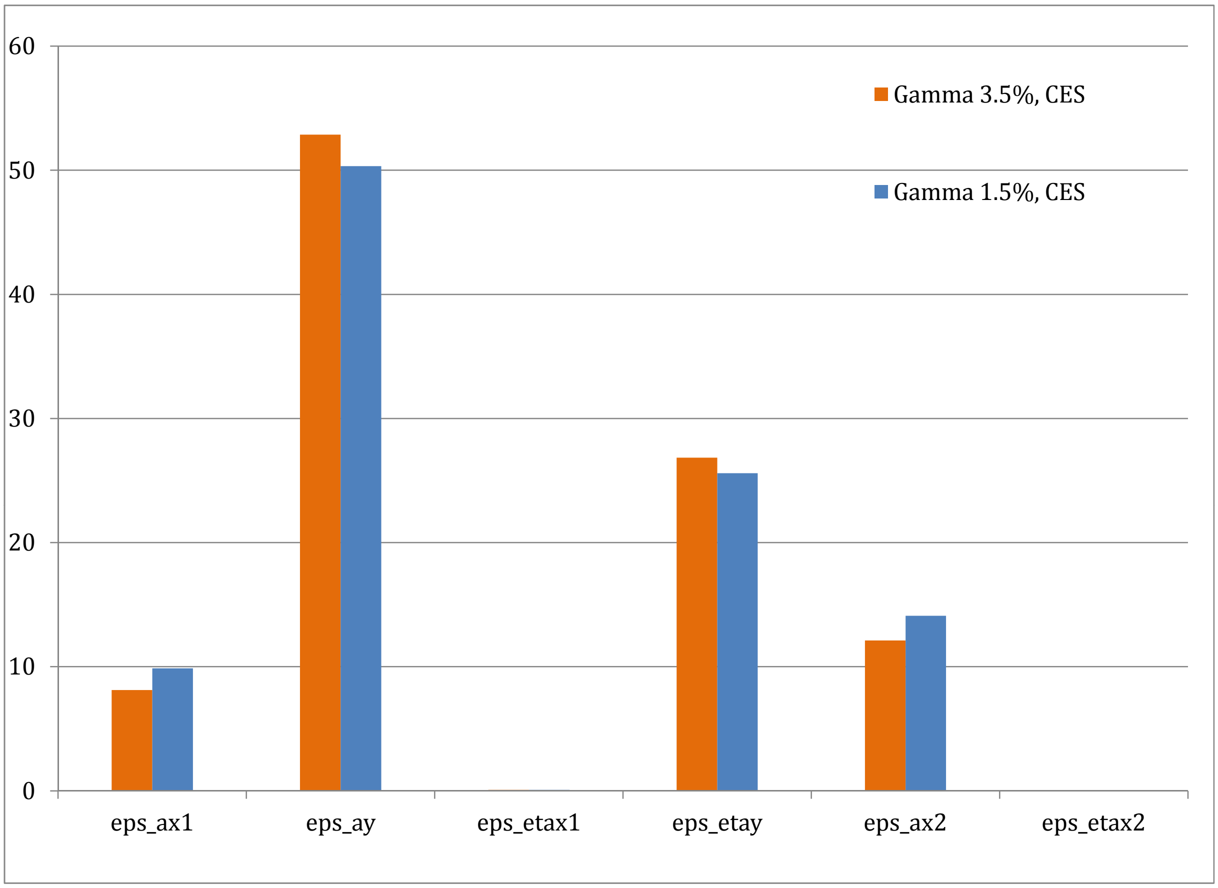


Figure 9: Alternative Aggregate Growth Rates

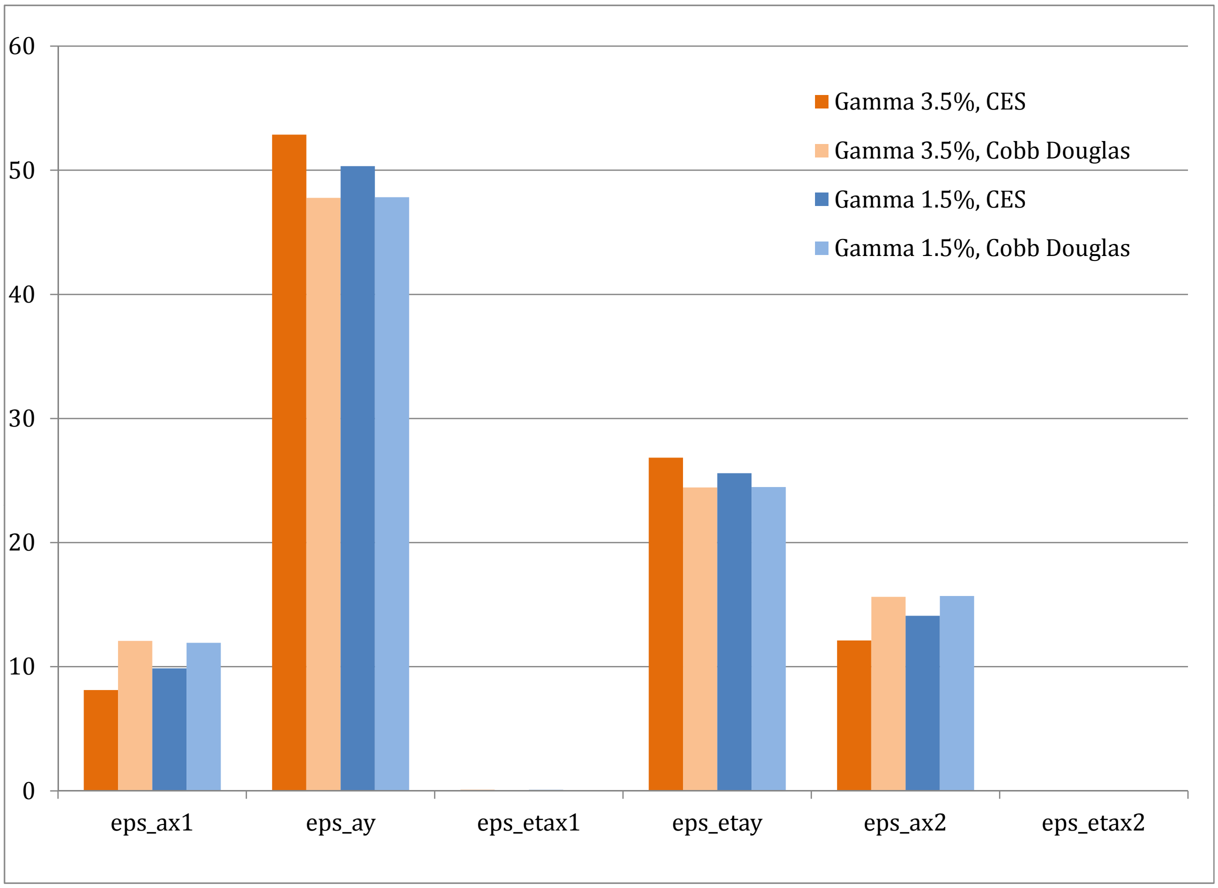


Figure 10: Alternative Aggregate Growth Rates, Constant and Flexible Cost Shares

13 Data Appendix

The following tables include a description of the sectors under analysis. Our definition of the equipment sector entails sectro 33 in the NAICS.

Industry/Commodity Description	NAICS
Agriculture, forestry, fishing and hunting	11
Crop production	111
Animal production	112
Forestry	1131, 1132
Logging	1133
Fishing, hunting and trapping	114
Support activities for agriculture and forestry	115
Mining	21
Oil and gas extraction	211
Coal mining	2121
Metal ore mining	2122
Nonmetallic mineral mining and quarrying	2123
Support activities for mining	213
Utilities	22
Electric power generation, transmission and distribution	2211
Natural gas distribution	2212
Water, sewage and other systems	2213
Construction	
Construction	23
Manufacturing	31, 32, 33
Animal food manufacturing	3111
Grain and oilseed milling	3112
Sugar and confectionery product manufacturing	3113
Fruit and vegetable preserving and specialty food manufacturing	3114
Dairy product manufacturing	3115
Animal slaughtering and processing	3116
Seafood product preparation and packaging	3117
Bakeries and tortilla manufacturing	3118
Other food manufacturing	3119
Beverage manufacturing	3121
Tobacco manufacturing	3122

Industry/Commodity Description	**NAICS**
Textile mills and textile product mills	313,314
Apparel manufacturing	315
Leather and allied product manufacturing, including footwear manufacturing	316
Sawmills and wood preservation	3211
Veneer, plywood, and engineered wood product manufacturing	3212
Other wood product manufacturing	3219
Pulp, paper, and paperboard mills	3221
Converted paper product manufacturing	3222
Printing and related support activities	323
Converted paper product manufacturing	3222
Printing and related support activities	323
Petroleum and coal products manufacturing	324
Basic chemical manufacturing	3251
Resin, synthetic rubber, and artificial synthetic fibers and filaments manufacturing	3252
Pesticide, fertilizer, and other agricultural chemical	3253
Pharmaceutical and medicine manufacturing	3254
Paint, coating, and adhesive manufacturing	3255
Soap, cleaning compound, and toilet preparation	3256
Other chemical product and preparation manufacturing	3259
Plastics product manufacturing	3261
Rubber product manufacturing	3262
Clay product and refractory manufacturing	3271
Glass and glass product manufacturing	3272
Cement and concrete product manufacturing	3273
Lime, gypsum and other nonmetallic mineral product	3274, 3279
Iron and steel mills and ferroalloy manufacturing	3311
Steel product manufacturing from purchased steel	3312
Alumina and aluminum production and processing	3313
Nonferrous metal (except aluminum) production	3314
Foundries	3315
Forging and stamping	3321
Cutlery and handtool manufacturing	3322
Architectural and structural metals manufacturing	3323
Boiler, tank, and shipping container manufacturing	3324
Hardware manufacturing	3325
Spring and wire product manufacturing	3326
Machine shops;	3327

Industry/Commodity Description	NAICS
Coating, engraving, heat treating, and allied activities	3328
Other fabricated metal product manufacturing	3329
Agriculture, construction, and mining machinery	3331
Industrial machinery manufacturing	3332
Commercial and service industry machinery	3333
Ventilation, heating, air-conditioning, equipment	3334
Metalworking machinery manufacturing	3335
Engine, turbine, and power transmission equipment	3336
Other general purpose machinery manufacturing	3339
Computer and peripheral equipment manufacturing	3341
Communications equipment manufacturing	3342
Audio and video equipment manufacturing	3343
Semiconductor and other electronic component	3344
Navigational, measuring, electromedica manufacturing	3345
Manufacturing and reproducing magnetic and optical media	3346
Electric lighting equipment manufacturing	3351
Household appliance manufacturing	3352
Electrical equipment manufacturing	3353
Other electrical equipment and component manufacturing	3359
Motor vehicle manufacturing	3361
Motor vehicle body and trailer manufacturing	3362
Motor vehicle parts manufacturing	3363
Aerospace product and parts manufacturing	3364
Railroad rolling stock manufacturing	3365
Ship and boat building	3366
Other transportation equipment manufacturing	3369
Household and institutional furniture and kitchen cabinet manufacturing	3371
Office furniture (including fixtures) manufacturing	3372
Other furniture related product manufacturing	3379
Medical equipment and supplies manufacturing	3391
Other miscellaneous manufacturing	3399
Wholesale trade	42
Wholesale trade	42

Industry/Commodity Description	NAICS
Retail trade	44, 45
Retail trade	44, 45
Transportation and warehousing	48, 49
Air transportation	481
Rail transportation	482
Water transportation	483
Truck transportation	484
Transit and ground passenger transportation	485
Pipeline transportation	486
Scenic and sightseeing transportation and support activities for transportation	487, 488
Couriers and messengers	492
Warehousing and storage	493
Information	
Newspaper, periodical, book, and directory publishers	5111
Software publishers	5112
Motion picture, video, and sound recording industries	512
Broadcasting (except internet)	515
Telecommunications	517
Data processing, hosting, related services, and other information services	518, 519
Finance and insurance	52
Monetary authorities, credit intermediation	521, 522
Securities, commodity contracts, and other invest	523
Insurance carriers	5241
Agencies, brokerages, and other insurance	5242
Funds, trusts, and other financial vehicles	525
Real estate and rental and leasing	53
Real estate	531
Automotive equipment rental and leasing	5321
Consumer goods rental and general rental centers	5322, 5323
Commercial and industrial machinery and equipment rental and leasing	5324
Lessors of nonfinancial intangible assets	533

Industry/Commodity Description	NAICS
Professional, scientific, and technical services	
Legal services	5411
Accounting, tax preparation, bookkeeping,	5412
Architectural, engineering, and related services	5413
Specialized design services	5414
Computer systems design and related services	5415
Management, scientific, and technical consulting	5416
Scientific research and development services	5417
Advertising and related services	5418
Other professional, scientific, and technical services	5419
Management of companies and enterprises	55
Management of companies and enterprises	55
Administrative and support	56
Office administrative services	5611
Facilities support services	5612
Employment services	5613
Business support services	5614
Travel arrangement and reservation services	5615
Investigation and security services	5616
Services to buildings and dwellings	5617
Other support services	5619
Waste management and remediation services	562
Educational services	61
Elementary and secondary schools	6111
Junior colleges, colleges, universities, prof schools	6112, 6113
Other educational services	6114-7
Health care and social assistance	62
Offices of health practitioners	6211, 6212, 6213
Home health care services	6216

Industry/Commodity Description	NAICS
Outpatient, laboratory, and other ambulatory care	6214, 6215, 6219
Hospitals	622
Nursing and residential care facilities	623
Individual and family services	6241
Community and vocational rehabilitation services	6242, 6243
Child day care services	6244
Arts, entertainment, and recreation	71
Performing arts companies	7111
Spectator sports	7112
Promoters of events, and agents and managers	7113, 7114
Independent artists, writers, and performers	7115
Museums, historical sites, and similar institutions	712
Amusement, gambling, and recreation industries	713
Accommodation and food services	72
Accommodation	721
Food services and drinking places	722
Other services (except public administration)	81
Automotive repair and maintenance	8111
Electronic and precision equipment repair	8112
Commercial and industrial machinery and equipment	8113
Personal and household goods repair	8114
Personal care services	8121
Death care services	8122
Drycleaning and laundry services	8123
Other personal services	8129
Religious organizations	8131
Grantmaking and giving services and social advocacy organizations	8132, 8133
Civic, social, professional, and similar organizations	8134, 8139
Private households	814
Postal Service	491