Monetary Policy Under Financial Exclusion*

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Abstract

We investigate the welfare implications of alternative monetary policy rules in a small open economy with access to world capital markets. Financial market access is costly and induces an endogenous segmentation of households into non-traders who never participate and traders who only participate intermittently in asset markets. The model can reproduce standard business cycle moments of open economies. Our main policy result is that inflation targeting outperforms both the monetary targeting and Taylor rule in this environment. Given widespread evidence of endemic financial exclusion throughout the world, these results suggest caution in importing monetary policy prescriptions tailored for developed countries into emerging economies.

1 Introduction

Policy interventions need to be justified by the existence of frictions in factor, goods or asset markets which prevent market allocations from achieving the first-best. Monetary policy is no exception. Thus, the standard macro models that underpin most modern monetary policy design rely on frictions in price adjustments to generate a role for monetary policy. Indeed, the wide-spread popularity of the Taylor Rule is at least partially based on the fact that it can be derived from a fully-specified structural model with sticky prices.

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Despite the popularity of sticky price models to study monetary policy, sticky prices are just one amongst many possible frictions in any economy. A different friction that is widely observed globally is financial exclusion wherein significant sections of the population do not participate in asset market transactions at all. While evidence on the extent of price rigidities across the world is somewhat limited, evidence on the degree of global financial exclusion is widespread with approximately 2.5 billion adults not using formal financial services at all. The friction is not uniquely a developing country phenomenon either. As late as in 1989, about 25 percent of US households had no checking accounts whatsoever while 59 percent did not have any interest bearing accounts!

When asset markets are segmented in this way monetary policy is no longer neutral. Since monetary policy is an asset market instrument, policy interventions redistribute purchasing power between those that participate in asset markets and those that don't. The real effects in these models arise due to their redistributive effects and have been explored by a number of authors starting with the Baumol-Tobin transactions demand model and its modern versions due to Jovanovic (1982), Romer (1986), Alvarez, Atkeson, and Kehoe (2002), Alvarez, Atkeson, and Edmond (2009), and Khan and Thomas (2015), amongst others.

The normative implications for monetary policy in models with segmented asset markets remain relatively unexplored. The only exception is Lahiri, Singh, and Vegh (2007) who showed that the standard Mundell-Fleming prescriptions for optimal exchange rate regimes get turned on their heads when the typical sticky price friction is replaced by a segmented asset market friction. How should monetary policy be structured when a significant segment of the population is excluded from asset markets? Does the optimal design of monetary policy in such environments resemble those derived from the sticky price paradigm? What are the welfare implications of different monetary policy rules when asset markets are segmented? How well does the Taylor rule do relative to the first-best? This paper examines these normative questions in an attempt to provide a more integrated perspective on optimal monetary policy that goes beyond the standard sticky price paradigm.

We develop a model of a small open economy with endogenously segmented asset markets to examine the effects of real and monetary shocks. It is an inventory-theoretic model of money where accessing asset markets involves both a sunk cost and a fixed cost. While access to asset markets is costly it is also useful as it allows households to smooth consumption. We show that the sunk cost of asset market access induces an endogenous separation of households into two types: non-trading and trading households. The fixed cost of shipping resources between goods and asset markets induces a further split of trading households into active and inactive households. The costs are calibrated to match the frequency of asset market activity and the share of permanently excluded households. Trading households optimally choose the time and size of portfolio rebalancing based on their cost and income realizations.

The paper studies alternative monetary rules such as money growth rules, inflation targeting rules and Taylor rules to compare their welfare outcomes with those under sticky prices. Using a calibrated version of the model we show that under productivity shocks an inflation targeting regime welfare dominates Taylor rules as it allows non-traders to smooth their consumption better than under Taylor rules. Lastly, we characterize the optimal monetary policy in this environment and contrast it with the Taylor rule.

2 Model

Consider a one good, small open economy inhabited by a continuum of households of measure one. Households have unrestricted access to world goods markets where the good trades at a fixed price of unity in terms of a numeraire world currency. The law of one price implies that good trades at the local currency price P which is also the exchange rate of the local currency against the numeraire currency. Cash will be demanded in this economy as we assume that local goods purchases require cash in advance.

Each household consists of a shopper-worker pair. The shopper carries out purchases of assets and goods while the worker supplies labor time to the market and earns wages. All households are endowed with one unit of labor time but they are heterogenous in the efficiency of their labor time. A fraction ω of households are low efficiency households with each unit of their labor time only producing $\epsilon < 1$ units of productive market labor time. The labor endowment of the remaining $1 - \omega$ fraction of households converts one-for-one to effective units of productive labor. The household type is public information.[1]

[1] The heterogeneity in the efficiency of labor will induce heterogeneity in incomes across households. This will be the key factor in some households choosing to remain permanently excluded from financial markets.

The economy is open to world capital markets where households can buy and sell international bonds denominated in terms of the good. The international bond pays a fixed real interest rate r. Hence, there are three ways of saving for households in this economy: international risk free bonds, a complete set of domestic state contingent nominal bonds, and money. However, access to capital markets is costly. Asset markets are physically separated from the goods market. Only households that pay the access costs can trade in financial securities. Households that choose not to access asset markets can potentially use inventories of nominal money holdings to save across periods. Money supply is determined by the government which alters money supply through open market operations in asset markets.

Households incur two types of access costs. First, they have to pay ζ (in units of the good) to open a brokerage account. This is a one-time payment and hence a sunk cost. Households with a brokerage account then have to pay a fixed cost ξ every period that they choose to make net transfers to or from their brokerage account. ξ is drawn every period from a distribution $G(\xi)$ that is independently and identically distributed across households and time. Lastly, opening a brokerage account also entails agreeing to a contract wherein a fraction $1 - \lambda$ of their income every period is directly deposited into the brokerage account leaving a fraction λ at home for use in the goods market.

This asset market structure is essentially the same as in Alvarez, Atkeson, and Kehoe (2002) and Khan and Thomas (2015). The loss of a fraction $1 - \lambda$ of wage earnings to the brokerage account generates a declining time path of household cash holdings which, in turn, necessitates a periodic rebalancing of the household cash holdings. The precise date at which a household chooses to rebalance its portfolio by shifting cash between the two markets depends, amongst other factors, on the realization of its fixed cost ξ.

The structure described above implies that households have to make two decisions regarding the extent of their participation in asset markets. First, they have to decide whether to participate at all. This decision entails comparing the sunk cost ζ of opening a brokerage account with the benefits from access to such an account. The benefits, as we shall formalize below will be the ability to use international capital markets to smooth out consumption even when income fluctuates. The gain from having a smoother consumption path would have to be adjusted by the fact that moving cash to and from the brokerage account is costly. This first level decision will bifurcate households into two groups – traders and non-traders. Second, in every period the trading households have to decide whether or not to rebalance their asset portfolio by moving cash between the brokerage account and home. This decision depends on a comparison of the gains from doing so with the household's cost realization ξ in that period.

The economy has four sets of actors: non-trading households, trading households, financial intermediaries and the government. Below we describe the periodic choices of each of these actors. Note that the endogenous sorting of households into traders and non-traders will be described later using the decision rules of each type of household. Throughout the paper we shall use $s^t = \{s_0, s_1, ..., s_t\}$ to denote the entire state history of aggregate realizations till date t and $f(s^t)$ to denote the probability density associated with this aggregate history. The date t realization of aggregate shock includes government policy variables for that period.

2.1 Non-trading households

Each non-trading household maximizes expected lifetime utility given by

$$V = \mathbb{E}\sum_{t=1}^{\infty} \beta^{t-1} u\left(c\left(s^t\right), h\left(s^t\right)\right) \tag{1}$$

where c is consumption, h is labor supply. The function u is assumed to be strictly increasing, concave, twice-differentiable in both arguments, and satisfies Inada conditions on c and h so that interior solutions are guaranteed.

The problem facing the non-trading households is simple. At the beginning of every period the household holds some cash balances from the previous period. The shopper takes this cash to the goods market where she uses the cash for buying the consumption good. The cash-in-advance constraint can be written as

$$M^N\left(s^{t-1}\right) \geq P\left(s^t\right) c^N\left(s^t\right)$$

where c^N denotes consumption of the non-trading household and M^N the non-trading household's beginning of period t cash holdings which is based on the state history s^{t-1}. The worker earns nominal wages $P\left(s^t\right) w^N\left(s^t\right) h^N\left(s^t\right)$ from his labor supply in period t. w^N denotes the real wage rate while h^N denotes hours of work supplied by the non-trading worker. The shopper and the worker meet up at the end of the day to consume the shopper's purchases during the day. This cash flow is not available to the shopper for purchasing goods in period t. As long as the cash-in-advance constraint binds, the worker's wage earnings in period t become the household's beginning of period cash next period. Thus,

$$M^N\left(s^t\right) = P\left(s^t\right) w^N\left(s^t\right) h^N\left(s^t\right)$$

Combining these two constraints gives

$$c^N\left(s^t\right) = \frac{w^N\left(s^{t-1}\right) h^N\left(s^{t-1}\right)}{\pi\left(s^t\right)} \equiv \frac{y^N\left(s^{t-1}\right)}{\pi\left(s^t\right)} \tag{2}$$

where $\pi\left(s^t\right) = \frac{P\left(s^t\right)}{P\left(s^{t-1}\right)}$ denotes the gross inflation rate between periods $t-1$ and t. Note that we are assuming here that the cash-in-advance constraint is binding for non-trading households. We shall ensure throughout by restricting parameters accordingly.

The household's labor supply decision is dynamic since work today only generates consumption tomorrow. The worker chooses sequences of labor supply to maximize expected lifetime welfare subject to

$$P\left(s^t\right) w^N\left(s^t\right) h^N\left(s^t\right) + M^N\left(s^{t-1}\right) \geq P\left(s^t\right) c^N\left(s^t\right) + M^N\left(s^t\right)$$

and equation (2). The non-trading household's optimal labor supply is then determined by the condition:

$$-u_2\left(c^N\left(s^t\right), h^N\left(s^t\right)\right) = \beta w^N\left(s^t\right) \mathbb{E}_t\left\{\frac{u_1\left(c^N\left(s^{t+1}\right), h^N\left(s^{t+1}\right)\right)}{\pi\left(s^{t+1}\right)}\right\} \tag{3}$$

This condition says that the optimal labor supply decision equates the marginal disutility from current labor supply with the expected marginal utility from the additional consumption tomorrow that is generated by the wage earnings from work today.

It is worth noting that for the cash-in-advance constraint to bind on all dates, we must have

$$u_1\left(c^N\left(s^t\right),h^N\left(s^t\right)\right)>\beta\mathbb{E}_t\left\{\frac{u_1\left(c^N\left(s^{t+1}\right),h^N\left(s^{t+1}\right)\right)}{\pi\left(s^{t+1}\right)}\right\}.$$

This expression when combined with equation (3) implies

$$w^N\left(s^t\right)>-\frac{u_2\left(c^N\left(s^t\right),h^N\left(s^t\right)\right)}{u_1\left(c^N\left(s^t\right),h^N\left(s^t\right)\right)}.$$

This will clearly hold in the steady state for $\frac{\beta}{\pi}<1$. Essentially, the constraint will therefore be binding as long as the economy is away from the Friedman rule. Note that if we define the nominal interest rate as $1+i=R\pi$ and impose the standard small open economy assumption of $R\beta=1$, then the Friedman rule, which is $i=0$, implies that $\frac{\pi}{\beta}=1$. For all $\frac{\pi}{\beta}>1$ we must have $i>0$. Hence, as long as $i>0$ the cash-in-advance constraint will bind.

2.2 Trading households

Each trading household that pays the sunk cost ζ to open a brokerage account solves

$$\max\sum_{t=1}^{\infty}\beta^{t-1}\int_{s^t}\int_{\xi^t}u\left(c^T\left(s^t,\xi^t\right),h^T\left(s^t,\xi^t\right)\right)dF\left(s^t\right)dG\left(\xi^t\right). \tag{4}$$

Note that optimal choices of trading households include not just the aggregate realizations but also the household specific history of the fixed cost shock ξ^t. The beginning of period t money balances of a trading household is

$$M^T\left(s^{t-1},\xi^{t-1}\right)=\lambda\left(s^{t-1}\right)P\left(s^{t-1}\right)w^T\left(s^{t-1}\right)h^T\left(s^{t-1},\xi^{t-1}\right)+A\left(s^{t-1},\xi^{t-1}\right) \tag{5}$$

where A denotes the money balances the household carried over from last period's money balances, i.e., the part of last period's money balances that was not spent on consumption purchases.

The brokerage account constraint for trading households is given by

$$R_tb\left(s^{t-1},\xi^{t-1}\right)+\frac{B\left(s^t,\xi^t\right)}{P\left(s^t\right)}+\left(1-\lambda\left(s^{t-1}\right)\right)w^T\left(s^{t-1}\right)h^T\left(s^{t-1},\xi^{t-1}\right)-b\left(s^t,\xi^t\right) \tag{6}$$

$$\geq \int_{s_{t+1}} \int_{\xi_{t+1}} q\left(s^t, s_{t+1}, \xi_{t+1}\right) \frac{B\left(s^t, s_{t+1}, \xi^t, \xi_{t+1}\right)}{P\left(s^t\right)} dF\left(s^{t+1}\right) dG\left(\xi^{t+1}\right) + \left[x\left(s^t, \xi^t\right) + \xi_t\right] I\left(s^t, \xi^t\right)$$

where x denotes the net transfer of funds from the brokerage account to home while I is an indicator function that takes the value one when the household is active in the asset market and zero otherwise. b denotes the risk-free international bond that pays an exogenous interest rate r so that $R_t = 1 + r_t$.

The goods market transactions of the trading household in every period have to obey the constraint

$$M^T\left(s^{t-1}, \xi^{t-1}\right) + P\left(s^t\right) x\left(s^t, \xi^t\right) I\left(s^t, \xi^t\right) \geq P\left(s^t\right) c^T\left(s^T, \xi^t\right) + A\left(s^t, \xi^t\right) \tag{7}$$

Equation (7) says that total consumption purchases and money balances carried over to the next period by the trading household cannot exceed the sum of money balances the household started the period with and the cash that it transferred from the brokerage account at the beginning of the period if it chose to be active that period. Lastly, the household transactions also have to obey a non-negativity constraint on excess money balances A that the household can carry across periods:

$$A\left(s^t, \xi^t\right) \geq 0 \tag{8}$$

Each trading household chooses sequences of $c^T, h^T, A, x, B,$ and b to solve (4) subject to equations (5), (6), (7), (8), and a bounded debt constraint.

As is standard, we also assume an initial period 0 before any aggregate or idiosyncratic shocks when trading households are all identical during which they can buy state-contingent domestic nominal bonds subject to the constraint

$$\bar{B}^T \geq \int_{s_1} \int_{\xi_1} q\left(s_1, \xi_1\right) B\left(s_1, \xi_1\right) dF\left(s_1\right) dG\left(\xi_1\right) \tag{9}$$

where $\bar{B}^T$ are the initial assets of trading households. Combining equation (9) with (6) gives

$$\bar{B}^T \geq \int_{s^t} \int_{\xi^t} q\left(s^t, \xi^t\right) \left\{\left(x\left(s^t, \xi^t\right) + P\left(s^t\right) \xi_t\right) I\left(s^t, \xi^t\right) - \left(1 - \lambda\left(s^{t-1}\right)\right) P\left(s^{t-1}\right) w^T\left(s^{t-1}\right) h^T\left(s^{t-1}\right)\right\} dF\left(s^t\right) dG\left(\xi^t\right) \tag{10}$$

Note that in deriving equations (9) and (10)we are assuming that initial foreign assets of trading households are zero.

2.3 Financial Intermediaries

Financial intermediaries in this economy issue domestic nominal state contingent bonds and buys government bonds. In addition, financial intermediaries also perform the role of brokerage agents for the financial transactions of households with respect to foreign assets. We follow Alvarez, Atkeson, and Kehoe (2002) in formalizing the intermediaries as entities that buy government bonds that are contingent on aggregate shocks and sell bonds to private agents whose payoffs are contingent on both aggregate and individual shocks.

Let $B(s^t, s_{t+1})$ denote nominal government bonds that pay one unit of domestic currency in state s_{t+1} under the history s^t. For each (s^t, s_{t+1}) financial intermediaries solve

$$\max \int_{\xi_{t+1}} \int_{\xi^t} q\left(s^t, s_{t+1}, \xi_{t+1}\right) B\left(s^t, s_{t+1}, \xi^t, \xi_{t+1}\right) dG\left(\xi^t\right) dG\left(\xi_{t+1}\right) - q\left(s^t, s_{t+1}\right) B\left(s^t, s_{t+1}\right)$$

subject to the constraint

$$\int_{\xi_{t+1}} \int_{\xi^t} B\left(s^t, s_{t+1}, \xi^t, \xi_{t+1}\right) dG\left(\xi^t\right) dG\left(\xi_{t+1}\right) \leq B\left(s^t, s_{t+1}\right)$$

The last constraint ensures that intermediaries can meet their obligations in all states for any given history.

2.4 Firms

The production side of the economy is characterized by a representative firm that operates in a perfectly competitive environment. It produces using the technology

$$y\left(s^t\right) = z\left(s^t\right) N\left(s^t\right)^{\nu}, \tag{11}$$

where N is a labor composite defined as

$$N \equiv \left[\left(H^T\right)^{1-\frac{1}{\sigma}} + \left(\varepsilon H^N\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where H^T and H^N denote (population weighted) total labor supply of trading and nontrading households, respectively, and $\varepsilon < 1$ is a relative efficiency parameter. This formulation allows the two types of labor to be imperfect substitutes with the parameter σ measuring the elasticity of substitution between the two types of labor ranging from perfect substitutes when $\sigma = \infty$ to Leontief preferences when $\sigma = 0$.[2] z is total factor productivity which is stochastic and follows a known first order autoregressive process.

Firms pay competitive real market wages (MPL), w^T and w^N , to traders and nontraders, respectively. Note that there will be a wage differential between traders and non-traders due to $\varepsilon < 1$. We assume that firms are owned by trading households with firm profits being rebated to the traders' brokerage account and home in the same proportions as their labor earnings.

2.5 Government

The government in this economy consists of a monetary authority and a fiscal authority. The fiscal authority issues bonds while the monetary authority issues money through open market operations and holds foreign reserves which earn the going world interest rate r. The consolidated budget constraint of the government is

$$P\left(s^t\right) R_t f\left(s^{t-1}\right) + \bar{M}\left(s^t\right) - \bar{M}\left(s^{t-1}\right) + \int_{s_{t+1}} q\left(s^t, s_{t+1}\right) B\left(s^t, s_{t+1}\right) dF\left(s_{t+1}\right) = P\left(s^t\right) f\left(s^t\right) + B\left(s^t\right) \tag{12}$$

where $\bar{M}$ denotes money supply. The left hand side of equation (12) gives the revenue stream of the consolidated government in any period with state history s^t while the right hand side gives the corresponding expenditure items. Note that the government also takes the world interest rate R as exogenously given to it.

The government also has some initial debt $\bar{B}$ which is distributed equally to all households that choose to be traders, i.e., those that pay the initial brokerage account fees of ζ. Hence,

$$\bar{B}^T = \frac{\bar{B}}{1-\omega}$$

[2] In our quantitative section below we shall examine the robustness of our main results to varying σ and show that our results are not too sensitive to σ.

where $1-\omega$ is the share of trading households in the economy. Clearly, ω is an endogenous variable that will be the outcome of choices made by individual households.

Clearly, the monetary authority has another policy choice to make, namely the exchange rate regime that it wants. If it wants to fix the exchange rate then its monetary policy would become endogenous while if it chooses to let the exchange rate to float freely then it retains autonomy over its monetary policy. If the monetary authority chooses the exchange rate then the domestic price level P becomes predetermined at all dates. We shall return to this in greater detail below when we describe alternative monetary policy rules.

2.6 Equilibrium

A competitive equilibrium in this economy is a sequence of quantities and prices:

$$\mathbf{Q} = \left\{c^T\left(s^t,\xi^t\right), c^N\left(s^t\right), h^T\left(s^t,\xi^t\right), h^N\left(s^t\right), b\left(s^t,\xi^t\right), A\left(s^t,\xi^t\right), B\left(s^t,s_{t+1},\xi^t\right), B\left(s^t,s_{t+1}\right), x\left(s^t,\xi^t\right), \omega\right\}$$
$$\mathbf{P} = \left\{P\left(s^t\right), q\left(s^t,s_{t+1}\right), q\left(s^t,s_{t+1},\xi_{t+1}\right), w^T\left(s^t\right), w^N\left(s^t\right)\right\}$$

such that for given distributions $G\left(s^t,s_{t+1}\right)$, $H\left(\xi^t\right)$ and sunk cost ζ, all households and firms satisfy their optimality condition and all markets clear at all dates and states at these prices and quantities.[3]

3 A Family Representation

The heterogeneity in the preceding problem can potentially become intractable due to high dimensionality of the associated state space. However, Khan and Thomas (2015) have shown that this problem can be recast in a simpler but equivalent problem of a family of traders that pools risk across households. This alternative family representation allows them to apply the methods of King and Thomas (2006) to solve the aggregate problem of trading households.

Three features of our assumed environment allow the family representation of the problem: (i) idiosyncratic fixed cost draws ξ; (ii) continuous access to the brokerage account for buying

[3] Note that we are imposing the restriction that aggregate shocks follow a Markov process. We shall maintain this restriction throughout the paper. Also, the aggregate state history s^t contains both aggregate productivity shocks as well as monetary policy shocks.

and selling bonds for traders;[4] and (iii) an initial period when households can acquire state-contingent bonds while being completely identical. The King-Thomas method uses the family construct to aggregate across member households by using the fact that individual households adopt (S,s) policy functions.

Khan and Thomas (2015) show that the traders' environment induces three effects: (i) at the beginning of any period t prior to the realization of the fixed cost shock ξ_t, all relevant heterogeneity between trading households is captured by their initial money balances $M\left(s^{t-1}, \xi^{t-1}\right)$. Hence, given $M\left(s^{t-1}, \xi^{t-1}\right)$ all optimal household plans are independent of history ξ^{t-1}; (ii) contingent on paying the fixed cost ξ_t in period t, the optimal plans of households are independent of their shock history ξ^t; and (iii) households follow threshold rules for accessing their brokerage account. A household elects to visit the brokerage account

$$V_0\left(s^t, \xi^t\right) - V_j\left(s^t, \xi^t\right) \geq \xi_t$$

where V_j denotes the value function of a household belonging to the j^{th} cohort. Khan and Thomas (2015) show this problem implies a threshold $\bar{\xi}^j$ such that all households of cohort j who receive draw a cost shock $\xi \leq \bar{\xi}^j$ pay the cost and become active in the brokerage account while those with cost realizations above the threshold stay inactive. Thus, given any initial money balance $M\left(s^{t-1}, \xi^{t-1}\right)$, there is a maximum fixed cost $\bar{\xi}^j$ that the household of cohort j is willing to pay in order to transfer resources from the brokerage account to home. Hence, the share of cohort j that becomes active at date t is

$$\alpha_t^j = G\left(\bar{\xi}^j\right)$$

The preceding description implies that all households that visit their brokerage firm in any period come out of the visit looking identical. They comprise a cohort of identical households. Thereafter, the only heterogeneity amongst households in this cohort is in the cost shocks that they receive. Since they all come out with identical nominal balances, they all have the same threshold level for their cost shock next period. As long as their cost realization is above

[4] Note that traders have perpetual access to brokerage account for buying bonds using funds that are already in the brokerage account. The restriction they face is on transferring resources between the brokerage account and home.

the threshold they remain inactive in the asset market next period. Hence, households that last visited the brokerage account together and have remained inactive continue to remain identical in terms of their nominal balance holdings in subsequent periods. In the event that one of these households receives a cost shock below their threshold they choose to visit the asset market. Households from other cohorts who receive a similarly favorable cost realization also visit the asset market. There they rebalance their nominal balances and return home as part of a new cohort with identical households.

This description leads directly to a representation of trading households at any point in time as being part of J distinct cohorts with J being endogenous to the distributions of the fixed transfer cost and the aggregate shocks. Following King and Thomas (2006) and Khan and Thomas (2015), we shall use θ_t^j to denote the measure of trading households in period t who last accessed their brokerage accounts j periods ago. In this formulation $j = \{1, .., J\}$ with $j = 1$ denoting the cohort that is active in the asset market in the current period while $j = J$ denotes the cohort that will become active in the current period with probability one since their nominal balances are so low that they would choose to become active even with the highest possible cost draw.

In the following we let α_t^j denote the probability of a household of cohort j becoming active in the brokerage market in period t. The cohort transitions are then summarized by

$$\theta_{t+1}^{j+1} = \left(1 - \alpha_t^j\right) \theta_t^j, \qquad 1 \leq j \leq J - 1 \tag{13}$$

$$\theta_{t+1}^1 = \sum_{j=1}^{J} \alpha_t^j \theta_t^j \tag{14}$$

Before stating the reformulated extended family problem it is useful to recast some of the aggregate variables in terms of the cohort-based notation described above. Thus, the total labor supply of trading households at any date t can now be written as:

$$H_t^T = \underbrace{\sum_{j=1}^{J} \alpha_t^j \theta_t^j}_{\equiv \theta_{t+1}^1} h_t^{0,T} + \sum_{j=1}^{J-1} \underbrace{\theta_t^j \left(1 - \alpha_t^j\right)}_{\equiv \theta_{1t+1}^{j+1}} h_t^{j,T};$$

$$= \theta_{t+1}^{1} h_t^{0,T} + \sum_{j=1}^{J-1} \theta_{t+1}^{j+1} h_t^{j,T},$$

with the last summation going up to only $J-1$ since $\alpha_{Jt} = 1$ for all t. Note that the labor supplied to firms $H_t^T = (1-\omega) h_t^T$ and $H_t^N = \omega h_t^N$.

Define the labor earnings of trading cohort $j-1$ that supplied $h_{t-1}^{j-1,T}$ and earned per unit wage w_{t-1}^T at date $t-1$ as

$$e_{t-1}^{j-1,T} \equiv h_{t-1}^{j-1,T} w_{t-1}^T$$

The total labor earnings of the trading *family* available for use at date t then is

$$e_t^T = \sum_{j=1}^{J} \theta_{t-1}^{j-1} e_{t-1}^{j-1,T}$$

Clearly, e_t^T is a state variable since these labor earnings only become available for use at date t. In addition, the profits rebated to the trading family equals

$$\rho_t \equiv \frac{y_{t-1} - (1-\omega) w_{t-1}^N H_{t-1}^N - \omega w_{t-1}^T H_{t-1}^T}{1-\omega}$$

Hence, the family's total and cohort-specific income (wage plus profit income) that is available for use at date t are, respectively,

$$\begin{aligned} y_t^T &\equiv e_t^T + \rho_t \\ y_t^{j,T} &= \left(1-\alpha_t^j\right) \theta_t^j e_{t-1}^{j-1,T} + \rho_t, \quad j = 1,..,J-1 \\ y_t^{0,T} &= \sum_{j=1}^{J} \alpha_t^j \theta_t^j e_{t-1}^{j-1,T} \end{aligned}$$

The extended trading family maximizes the cohort-share weighted welfare of all households in the family. Letting $a = \frac{A}{P}$ denote real balances carried over from the previous period's opening balances, we can write the family's value function as

$$V\left(\left\{\theta_t^j, a_t^j, e_t^j\right\}_{j=1}^{J}, b_t, y_t^T, m_t, z_t, \lambda_t, \pi_t\right) = \max[\sum_{j=1}^{J} \theta_t^j \left(\alpha_t^j u\left(c_t^{0T}, h_t^{0T}\right) + \left(1-\alpha_t^j\right) u\left(c_t^{jT}, h_t^{jT}\right)\right) \tag{15}$$

$$+\beta E\left[V\left(\left\{\theta_{t+1}^{j},a_{t+1}^{j},e_{t+1}^{j}\right\},b_{t+1},y_{t+1}^{T},m_{t+1},z_{t+1},\lambda_{t+1},\pi_{t+1}\right)\right]$$

π depends on monetary policy and endogenously determines m'. The family knows its law of motion. Finally, z and λ follow AR(1) processes.

The family maximizes weighted lifetime family welfare subject to the constraints

$$\left(m_t-\frac{m_{t-1}}{\pi_t}\right)+(1-\lambda_t)\frac{y_t^T}{\pi_t}+\left(R(b_t)\,b_t-b_{t+1}\right) \tag{16}$$

$$=\sum_{j=1}^{J}\theta_{jt}\alpha_{jt}\left(c_t^{0T}+a_{t+1}^{1}-\frac{a_t^j}{\pi_t}-\lambda_t\frac{y_t^{jT}}{\pi_t}\right)+\sum_{j=1}^{J}\theta_t^j\int_0^{G^{-1}\left(\alpha_t^j\right)}\xi dG(\xi)$$

$$\frac{a_t^j}{\pi_t}+\frac{\lambda_t y_t^{jT}}{\pi_t}=c_t^{jT}+a_{t+1}^{j+1},\qquad j=1,...,J-1 \tag{17}$$

$$\theta_t^j\left(1-\alpha_t^j\right)=\theta_{t+1}^{j+1},\qquad j=1,...J-1 \tag{18}$$

$$\theta_{t+1}^{1}=\sum_{j=1}^{J}\theta_t^j\alpha_t^j. \tag{19}$$

Equation (16) is the brokerage account constraint for the family at date t. A household belonging to cohort j becomes active in the asset market if they get a transfer cost realization ξ that is lower than their threshold $\bar{\xi}^j$. Hence, the total fixed payments made by different cohorts becoming active at t is $\sum_{j=1}^{J}\theta_t^j\int_0^{G^{-1}\left(\alpha_t^j\right)}\xi dG(\xi)$. Note that $\alpha_t^J=1$ and $a_{t+1}^J=0$ which follows from the cost restriction that ensures that visiting the brokerage account is always optimal for cohort J. Equation (17) is the flow budget constraint for goods market transaction for the different cohorts while equations (18) and (19) give the evolution of the measure of the different cohorts. $R(b_t)$ is the exogenously given interest rate function with $R'\leq 0$.[5] We describe the first order conditions for this problem in Appendix 7 of the paper.

In a competitive equilibrium, firms and households take wages and prices as well as their laws of motion as given. Given exogenous shock processes, households and firms solve their optimization problems. Any candidate equilibrium then requires that firms pay workers their

[5]We introduce this specification to endogenously determine the steady state asset position of the economy. As is well known, in the absence of such an assumption, small open economy models like our's generically have a unit root.

marginal product and the aggregate resource constraint for the economy is satisfied:

$$y_t + \omega \left[R\left(b_t\right) b_t - b_{t+1} \right]$$
$$= \omega c_t^N + (1-\omega) \left(\sum_{j=1}^{J} \alpha_t^j \theta_t^j c_t^{0T} + \sum_{j=1}^{J-1} \left(1 - \alpha_{tjt}^j\right) \theta_t^j c_t^{jT} + \sum_{j=1}^{J} \theta_t^j \int_0^{G^{-1}\left(\alpha_t^j\right)} \xi dG\left(\xi\right) \right)$$

4 Quantitative illustration

The model specified above is very rich in terms of the agent heterogeneity it accommodates. That however implies that to understand and illustrate the dynamic behavior of the variables in the model we need to numerically simulate the model. In this section we quantitatively examine the response of the key macroeconomic variables to exogenous shocks to productivity and monetary policy.

We assume that the utility function of all agents is

$$u\left(c, h\right) = \frac{c^{1-\sigma} h^{\gamma}}{1-\sigma}, \quad \sigma > 0,\ 0 \leq \gamma < 1$$

We set $\sigma = 2$. These preferences imply that labor supply will be a function of household wealth in addition to wages. Since the labor productivity of non-trading households is a fraction ε of the corresponding productivity of trading households, we calibrate γ and ε to target a steady state $\frac{c^N}{c^T} = 0.5$ and $h^T = h^N = 1/3$.[6] This yields $\gamma = -1.224$ and $\varepsilon = 0.49$.

We assume that the stochastic productivity process is given by

$$\log z_t = \rho \log z_{t-1} + \sigma_z \epsilon_t, \quad \sigma_z = 0.0025 \tag{20}$$

The disturbance term ε is assumed to be i.i.d. and is drawn from $N\left(0,1\right)$. We should note that $\sigma_z = 0.0025$ implies a standard deviation of output of only 0.7 percent. We vary this later to examine more volatile output processes.

In order to examine the effects of asset market disturbances we allow the share of traders'

[6] An alternative and popular specification for preference is $u\left(c - g\left(h\right)\right)$. This specification implies that labor supply is only a function of the wage rate. Since non-traders have a lower labor productivity their wages will also be correspondingly lower than traders. As a consequence, these alternative preferences would imply lower relative labor supply by non-traders.

income that is deposited directly in their brokerage account, λ, to be stochastic with

$$\lambda_t = \bar{\lambda} + \sigma_\lambda \varsigma_t, \quad \varsigma_t \sim N(0,1) \tag{21}$$

where ς is i.i.d. and is drawn from a standard normal distribution. For our baseline experiments, we shut down this margin by setting $\sigma_\lambda = 0$. We interpret shocks to λ as velocity shocks since the amount of cash in the household relative to its income changes when λ changes.

There are two types of costs in the model associated with accessing the brokerage account. The first is the one-time sunk cost ζ that is paid at date $t = 0$ to open a brokerage account. We pick this parameter to target $\omega = 0.6$ which is the proportion of non-traders in the economy. The second cost is the fixed transfer cost parameter ξ. We assume that this is i.i.d. and drawn from a uniform distribution over the interval $[0, 0.25]$. This distribution and its support are key in determining the number of steady state cohorts J. The assumed distribution implies $J = 6$.

We choose the discount factor β to target steady state real interest of 3 percent while the monetary policy processes described below are all calibrated to target a 3 percent steady state inflation rate. The steady state real interest rate and inflation rate along with the assumed distribution for ξ imply an average duration between account transfers of 4.5 quarters. These numbers are in line with those in Khan and Thomas (2015).

Table 1 shows the steady state distribution of cohorts and their characteristics under this parameterization with 3% annual inflation rate, 3% real interest rate and $J = 6$. The table illustrates some key aspects of optimal behavior by trading households. First, the third row (for α^j) shows that the probability of becoming active in the brokerage market is rising in the time since the cohort was last active in asset markets. This implies that the threshold cost $\bar{\xi}^j$ is rising as j rises so that the households belonging to cohort $j = 6$ become active even at the highest cost realization. Hence $\alpha^6 = 1$. Second, the real balances carried over across periods by inactive trading families is declining in the time since they were last active. This can be seen in the row for a^j where the numbers secularly decline with $a^6 = 0$ implying that the cohort who had last visited the asset market six periods back runs its excess real balances to zero since it knows that it will visit the brokerage account with probability one the next period. This is the Ss feature of the solution to the household's cash inventory management problem.

A third feature that is noteworthy is that while consumption declines with time since the last asset market visit by the household, labor supply actually rise till period 5 before falling sharply in the last period. The path for labor supply partly reflects the fact that households can undo the constraint of a declining stock of household real balances by working more hours and generating additional cash holding next period through the part of the earning that they retain at home. Households clearly do that for the first five periods after they last visited the brokerage market. For $j = 6$ cohort however, the incentive to work more in order to generate additional cash balances for tomorrow is missing since they visit the brokerage market with certainty next period. Consequently, their labor supply declines sharply in that last period.

Table 1. Steady state cohort characteristics of traders

Time since active: j	1	2	3	4	5	6
Starting populations: θ^j	0.21	0.207	0.197	0.173	0.134	0.079
Fraction active: α^j	0.013	0.051	0.119	0.226	0.41	1
Consumption: c^{jT}	1.0	0.997	0.99	0.98	0.965	0.89
Labor supply: h^{jT}	0.336	0.337	0.337	0.338	0.339	0.297
Real balances: a^j	1.88	1.46	1.057	0.67	0.29	0

Note. The table reports the characteristics of the different cohorts of traders in steady state.

4.1 Monetary Policy

We study the effects of monetary policy rules in this environment by comparing outcomes four different policy regimes. We shall study the behavior of the key variables in the model under each of the rules and then compare welfare under the different rules. To aid our discussion of alternative monetary rules it is convenient to define the domestic nominal interest rate i as

$$1 + i_t = \mathbb{E}_t R_t \pi_{t+1} \tag{22}$$

Recall that R is the gross real interest factor and π is the gross inflation factor.

The four different monetary rules we study are:

1. **Fixed monetary growth rule:**

$$\bar{M}_{t+1} = \mu_{ss}\bar{M}_t \tag{23}$$

where we set $\mu_{ss} = 0.03$ in order to target a 3 percent steady state inflation rate.

2. **Cyclical money growth rule:**

$$\bar{M}_{t+1} = \mu_t \bar{M}_t \tag{24}$$

$$\frac{\mu_t}{\mu_{ss}} = \left(\frac{Y_t}{Y_{ss}}\right)^{\alpha_m} \tag{25}$$

where μ_{ss} is the constant steady state rate of money growth, Y denotes domestic output at date t, Y_{ss} is the steady state level of output and α_m is a constant. Clearly, $\alpha_m > 0$ corresponds to a procyclical money growth rule while $\alpha_m < 0$ is a countercyclical money growth rule. When $\alpha_m = 0$ the cyclical rule collapses to the fixed money growth rule. For the procyclical rule we assume $\alpha_m = 0.75$ and for the countercyclical rule we set $\alpha_m = -0.75$.

3. **Inflation targeting:**

$$\pi_t = \pi^* \tag{26}$$

We set the inflation target $\pi^* = 0.03$ which implies a steady state inflation rate of 3 percent.

4. **Taylor rule:**

$$i_t - i^* = \alpha_y (Y_t - Y_{ss}) + \alpha_\pi (\pi_t - \pi^*) \tag{27}$$

where i^* is the steady state nominal interest. We set $\alpha_y = 0.3$ and $\alpha_\pi = 1.5$ in accordance with standard estimates for these two parameters for the USA.

One caveat regarding the Taylor rule in our framework is warranted. Given our open economy environment, the interest parity condition always holds. Since we have normalized world inflation to zero, the world nominal interest rate equals the world real interest rate. this implies that

$$1 + i_t = \mathbb{E}_t R_t \varepsilon_{t+1}$$

where ε is the rate of depreciation of the local currency. The law of one price along with zero world inflation implies $\pi_t = \varepsilon_t$ for all t. Using this along with our maintained assumption $R^* \beta = 1$, it is easy to check that the interest parity condition implies that

$$i_t - i^* = R_t - \beta^{-1} + \mathbb{E}_t (\pi_{t+1}) - \pi^* + r_t (\mathbb{E}_t (\pi_{t+1}) - 1) - \beta^{-1} (\pi^* - 1)$$

For small π, π^*, r and large β, this expression is well approximated by

$$i_t - i^* = R_t - \beta^{-1} + \mathbb{E}_t (\pi_{t+1}) - \pi^*$$

Combining the Taylor rule with this interest parity condition then gives

$$\mathbb{E}_t (\pi_{t+1}) = \alpha_y (y_t - y^*) + \alpha_\pi (\pi_t - \pi^*) - R (b_{t+1}) + \beta^{-1} + \pi^* \quad (28)$$

Equation (28) is an expectational linear difference equation that must hold for consistency in an open economy in which the monetary authority follows a Taylor rule and also permits unrestricted capital mobility so that interest parity holds.

5 Results

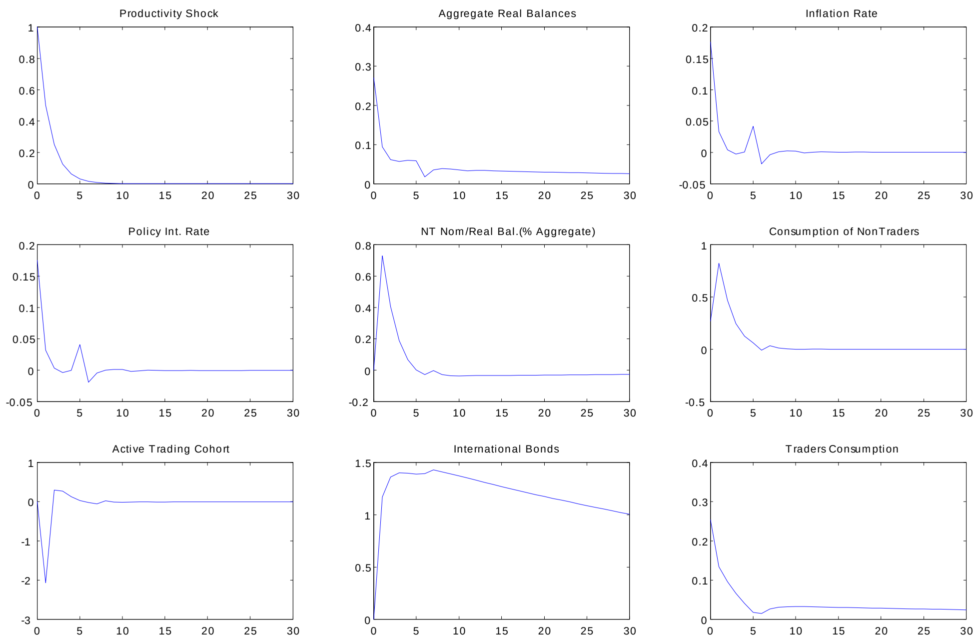
To illustrate the dynamic behavior of the model we focus on the macroeconomic effects of a one standard deviation positive shock to productivity drawn from the process given in equation (20). Since the responses are dependent on the specific policy environment, we present three sets of results, each corresponding to a different monetary rule.

5.1 The Endowment Economy

We start by describing the dynamic impulse responses in the case where there is no endogenous labor supply and output is exogenous. These results then provide a useful benchmark to evaluate the effects in the production economy case. The shock we consider is a shock to output that follows the same process as the shock to productivity specified in equation (20).

The first set of impulse responses are when the monetary authority maintains a fixed money growth rate given as summarized in equation (23). Figure 1 shows the impulse response

Figure 1: Fixed money growth: Impulse responses to productivity shock in the endowment economy



Note: The figure gives the impulse responses of macroeconomic variables to a one standard deviation positive shock to productivity parameter z in the endowment economy version of the model when the money growth rate is constant at 3 percent.

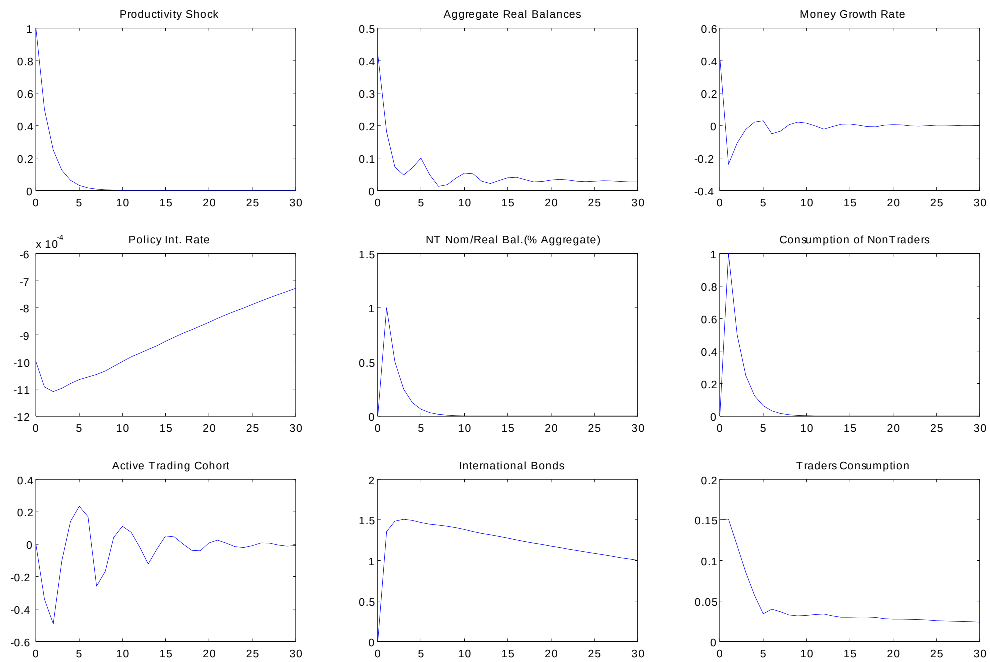
Figure 2 shows the impulse responses to a productivity shock of the same variables but when the monetary authority follows an inflation targeting rule given by equation (26).

Lastly, Figure (3) shows the impulse responses to a productivity shock in the endowment economy model when the monetary authority follows the Taylor rule as given in equation (27).

The three sets of impulse responses suggest similarity in outcomes under the three regimes. First, the higher output causes a decline in the share of households who choose to become active at any date implying that the threshold transfer cost falls in response to the shock. Intuitively, households realize that they will have additional money balances tomorrow which reduces their perceived benefits from visiting the asset market to re-balance their cash portfolio.

Second, both non-trading and trading households increase their consumption in response

Figure 2: Inflation targeting: Impulse responses to productivity shock



Note: The figure gives the impulse responses of macroeconomic variables to a one standard deviation positive shock to productivity parameter z in the endowment economy version of the model when the central bank target 3 percent inflation.

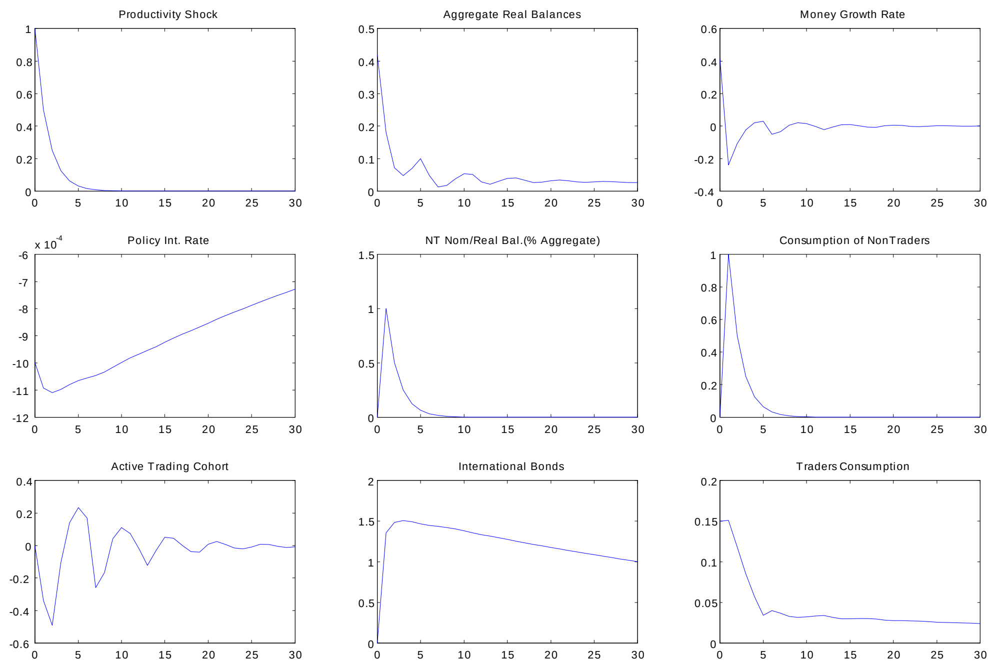
to the positive income shock. Trading households respond less however since they smooth out the income shock by saving abroad in foreign bonds. This shows up in a rise in the economy's foreign bond holdings.

Third, aggregate real balances rise in all cases. In the case of a fixed money growth rule, inflation rises as a result. Under inflation targeting as well as the Taylor rule the monetary authority responds by lowering the rate of money growth in order to lower inflation back down towards its target level.

5.2 The Production Economy

We next turn to the production economy case where labor is endogenously chosen and consequently output is endogenous as well. As in the endowment economy case, we present impulse responses for three different monetary policy frameworks. Figure (4) shows the impulse responses of the macroeconomic variables in the model to a productivity shock when

Figure 3: Taylor rule: Impulse responses to productivity shock in the endowment economy



Note: The figure gives the impulse responses of macroeconomic variables to a one standard deviation positive shock to productivity parameter z in the endowment economy version of the model when the central bank follows a Taylor rule.

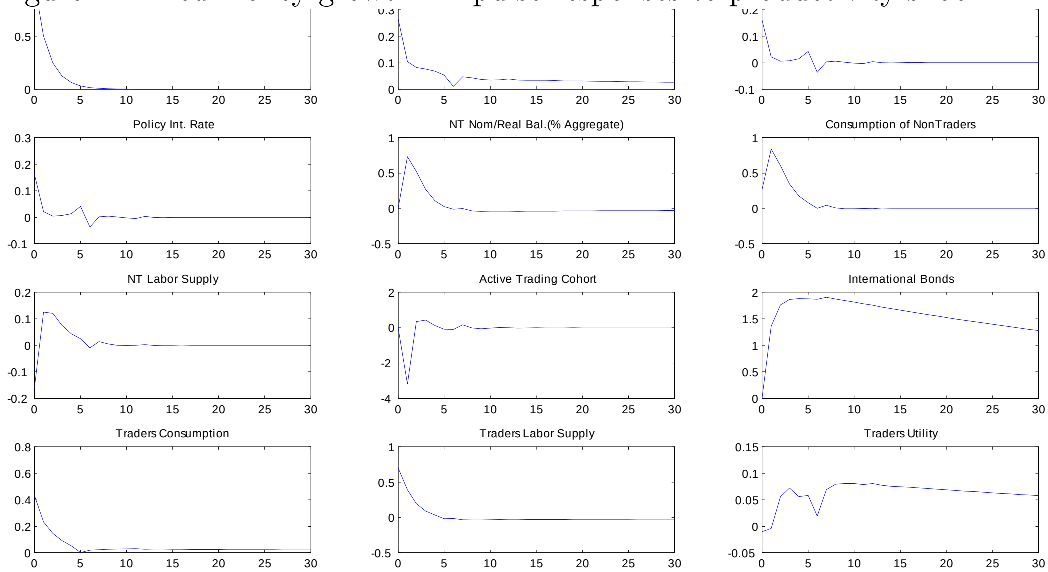
the monetary authority follows a fixed money growth rule.

Figure 5 shows the impulse responses to a productivity shock of the same variables but when the monetary authority follows the inflation targeting rule specified in equation (26).

Finally, Figure (6) shows the impulse responses to a productivity shock in the production economy when the monetary authority follows the Taylor rule in equation (27).

All three regimes reveal similar responses to productivity shocks. The main difference though with respect to the endowment economy case is that now households have an extra avenue for self-insurance: they can vary their labor supply to the market in response to productivity shocks, thereby potentially offsetting the fect of the productivity shock on their incomes and consequently on consumption. This can be clearly seen from the decline in the labor supply by non-trading households in response to a positive productivity shock. Since these households cannot store their temporarily high incomes by using asset markets they self-insure by just reducing their labor supply and instead enjoy greater leisure during the

Figure 4: Fixed money growth: Impulse responses to productivity shock



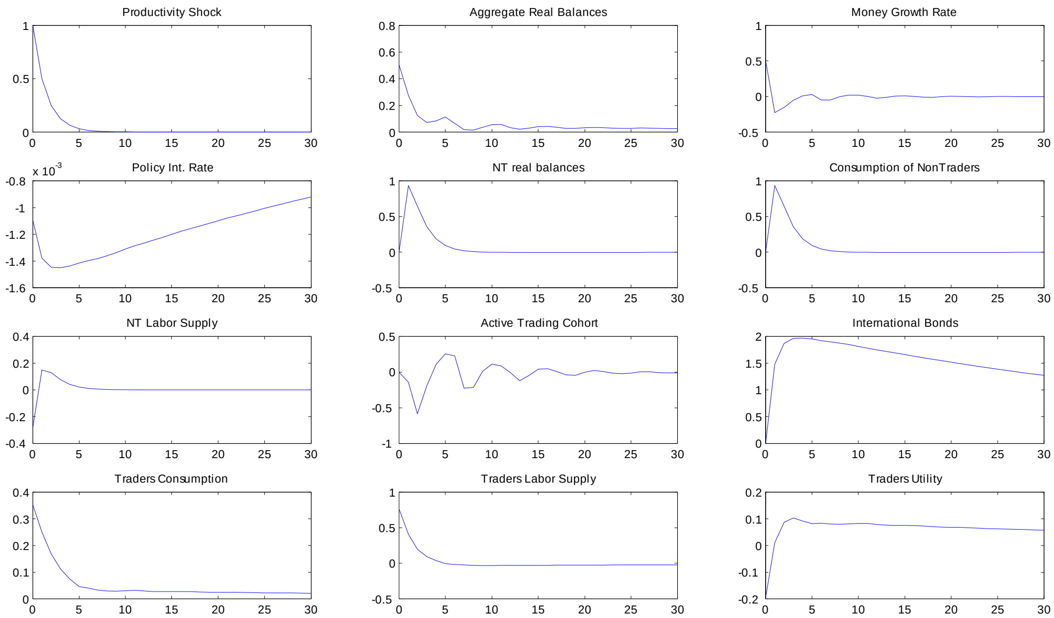
Note: The figure gives the impulse responses of macroeconomic variables to a one standard deviation positive shock to productivity parameter z in the production economy version of the model when the money growth rate is constant at 3 percent.

temporarily high productivity period. Trading households on the other hand take advantage of the high productivity peruod by supplying more labor and storing their temporarily high incomes in the form of international bonds. This is similar to the endowment economy case we described earlier.

6 Conclusions

In this paper we have evaluated the positive and normative effects of alternative monetary policy rules in a small open economy in which a subset of households engage in financial market transactions. Costly access to financial market induces an endogenous segmentation of households into non-traders who never participate and traders who only participate intermittently in asset markets. Our model generates an endogenous distribution of housholds such that only high income households choose to access financial markets. Households with incomes below a threshold level choose to be non-traders and consequently remain outside

Figure 5: Inflation targeting: Impulse responses to productivity shock

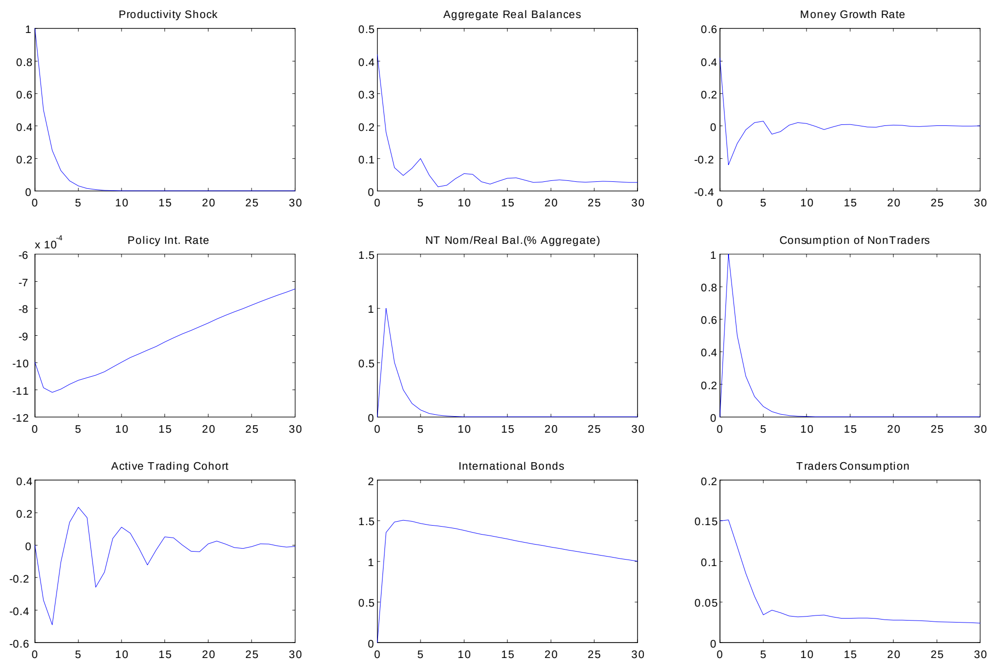


Note: The figure gives the impulse responses of macroeconomic variables to a one standard deviation positive shock to productivity parameter z in the production economy version of the model when the central bank target 3 percent inflation.

financial markets.

We have studied monetary policy rules in this environment by comparing outcomes under four different policy regimes: (i) a fixed money growth rule, (ii) a cyclical (both pro- and counter-) money growth rule, (iii) inflation targeting, and (iv) Taylor rule. We rank welfare under these rules in an endowment as well as a production economy. Under productivity shocks, we show that an inflation targeting regime welfare dominates Taylor rules as it allows non-traders to smooth their consumption better than under Taylor rules. Our results provide additional support for inflation targeting since these results are generated from an environment with endogenously segmented asset markets which is very different from the sticky price friction that typically underlies the standard Taylor Rule prescription in modern central banking theory.

Figure 6: Taylor rule: Impulse responses to productivity shock



Note: The figure gives the impulse responses of macroeconomic variables to a one standard deviation positive shock to productivity parameter z in the production economy version of the model when the central bank follows a Taylor rule.

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7 Appendix: Traders' first order conditions

In the following we denote the multipliers on constraints (16), (17), (18) and (19) as ν, ϕ^j, ψ^j, and δ, respectively. The first order conditions of the trading family with respect to labor supply choice for each cohort h_{jt} are given by

$$\mathbb{E}_t \left\{ \frac{\beta w_t}{\pi_{t+1}} \left(u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) + \lambda_{t+1} \frac{\theta_{t+2}^2}{\theta_{t+1}^1} \left[u_1\left(c_{t+1}^{1T}, h_{t+1}^{1T}\right) - u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) \right] \right) \right\} = -u_2\left(c_t^{0T}, h_t^{0T}\right)$$

$$\mathbb{E}_t \left[\frac{\beta w_t}{\pi_{t+1}} \left(u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) + \lambda_{t+1} \frac{\theta_{t+1}^{j+2}}{\theta_{t+1}^{j+1}} \left[u_1\left(c_t^{j+1,T}, h_{t+1}^{j+1,T}\right) - u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) \right] \right) \right] = -u_2\left(c_t^{jT}, h_t^{jT}\right), j = 1, ., J -$$

$$\mathbb{E}_t \left[\frac{\beta w_t}{\pi_{t+1}} u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) \right] = -u_2\left(c_t^{J-1,T}, h_t^{J-1,T}\right)$$

The optimality conditions with respect to c_t^j are

$$\begin{aligned} \nu_t &= u_1\left(c_t^{0T}, h_t^{0T}\right) \\ \theta_{t+1}^{j+1} u_1\left(c_t^{jT}, h_t^{jT}\right) &= p_t \psi_{jt}; \quad j = 1, ..J-1 \end{aligned}$$

The first order conditions with respect to the cohort shares θ_{t+1}^{j+1} are

$$\delta_t = \beta \mathbb{E}_t \left[\begin{array}{c} \left(\alpha_{t+1}^1 u\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) + \left(1-\alpha_{t+1}^1\right) u\left(c_{t+1}^{1T}, h_{t+1}^{1T}\right)\right) \\ -\upsilon_{t+1}\alpha_{t+1}^1 \left(c_{t+1}^{0T} + a_{t+2}^1 - \frac{a_{t+1}^1}{\pi_{t+1}} - \lambda_{t+1}\frac{y_{1t+1}^T}{\pi_{t+1}}\right) \\ -\nu_{t+1}\frac{\left(\alpha_{t+1}^1\right)^2 \xi_h}{2} + \frac{\nu_{t+1}}{\pi_{t+1}}\left(1-\lambda_{t+1}\right) e_{t+1}^1 \\ +\phi_{t+1}^1\left(1-\alpha_{t+1}^1\right) + \delta_{t+1}\alpha_{t+1}^1 \end{array} \right],$$

$$\phi_t^j = \beta \mathbb{E}_t \left[\begin{array}{c} \left(\alpha_{t+1}^{j+1} u\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) + \left(1-\alpha_{j+1,t+1}\right) u\left(c_{t+1}^{j+1,T}, h_{t+1}^{j+1,T}\right)\right) \\ -\upsilon_{t+1}\alpha_{t+1}^{j+1} \left(c_{t+1}^{0T} + a_{t+2}^1 - \frac{a_{t+1}^{j+1}}{\pi_{t+1}} - \lambda_{t+1}\frac{y_{t+1}^{j+1,T}}{\pi_{t+1}}\right) \\ -\nu_{t+1}\frac{\left(\alpha_{t+1}^{j+1}\right)^2 \xi_h}{2} + \frac{\nu_{t+1}}{\pi_{t+1}}\left(1-\lambda_{t+1}\right) e_{t+1}^{j+1} \\ +\phi_{t+1}^{j+1}\left(1-\alpha_{t+1}^{j+1}\right) + \delta_{t+1}\alpha_{t+1}^{j+1} \end{array} \right],$$

$$j = 1, ..J-1; \quad \phi_{Jt} = 0$$

Optimal family choices for a_{jt+1} must satisfy

$$u_1\left(c_t^0, h_t^0\right) = \mathbb{E}_t\left[\frac{\beta}{\pi_{t+1}}\left(\theta_{t+1}^1 \alpha_{t+1}^1 u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) + \theta_{t+1}^2 u_1\left(c_{t+1}^{1T}, h_{t+1}^{1T}\right)\right)\right]$$

$$u_1\left(c_t^{jT}, h_t^{jT}\right) = \mathbb{E}_t\left[\frac{\beta}{\pi_{t+1}}\left(\alpha_{t+1}^{j+1} u_1\left(c_{t+1}^{0T}, h_{t+1}^{0T}\right) + \left(1-\alpha_{t+1}^{j+1}\right) u_1\left(c_{t+1}^{j+1,T}, h_{t+1}^{j+1,T}\right)\right)\right],\ j = 1..J-1$$

The optimal choice of the share of cohort j becoming active at date t, α_{jt}, is governed by the condition

$$u\left(c_t^{0T}, h_t^{0T}\right) - u\left(c_t^j, h_t^{jT}\right) - \nu_t\left[\left(c_t^{0T} + a_{t+1}^1 - \frac{a_t^j}{\pi_t} - \lambda_t \frac{y_t^{jT}}{\pi_t}\right) + \alpha_t^j \xi_h\right]$$

$$= \phi_t^j - \delta_t, \quad j = 1, ..J-1; \;\; \alpha_J = 1$$

Finally, the first order condition with respect to b_{t+1} is

$$\nu_t = \beta R'\left(b_{t+1}\right) \mathbb{E}_t\left[\nu_{t+1}\right].$$