Real Exchange Rates and Commodity Prices*
(Preliminary and Incomplete)

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Abstract

We show that a small number of commodity prices can explain a large fraction of the volatility in bilateral real exchange rates between developed economies. We analyze the real exchange rates between the United States and Germany, Japan and the United Kingdom. We show that with up to six commodities we can explain between 50% and 80% of the volatility of those three real exchange rates.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
1 Introduction

In this paper we show that a small number of commodity prices can explain a substantial fraction of the movements in real exchange rates (RER) among industrialized countries. Specifically, we study the behavior of the RER of Japan, the United Kingdom and Germany against the United States for the 1960-2010 period. A rough summary of the results is that with 10 commodity prices, we can explain around 50% of the volatility of the RER of the United Kingdom and Germany, and around 84% of the volatility of the RER of Japan.

We find these results remarkable, given the difficulty the literature has encountered to relate RER movements across developed economies to fundamentals, as has been widely documented, for example in Betts and Kehoe (2004) and Engel (1999). This is in contrast with the ability of small open economy models of commodity producing countries, as shown in Chen and Rogoff (2003) and more recently by Hevia and Nicolini (2013), which show how changes in the international price of the exportable commodity of a small open economy induces changes in the corresponding RER. As we show, that same idea can go a long way in explaining movements in RER among developed economies.

We spell out a totally standard model that makes explicit the production of commodities and the use of commodities in the production of manufactures. In particular, we generalize the model used in Hevia and Nicolini. Commodities are typically ignored in two-country models of international trade. We guess this is because commodity production is associated to developing economies. However, the numbers on commodity world trade are far from trivial: Total trade in just a few commodities (34) accounts for 20% of total world trade. This number clearly underestimates the true share of commodities, since when aluminum is exported, it is fully counted as a manufactured good, while an important component of its cost depends on the price of iron. The same happens when a car is exported.

The idea we exploit in the paper is very simple: fluctuations in the prices of commodities
affect manufacturing costs, and therefore manufacturing prices, which in turn induce changes in final good costs. These cost fluctuations translate in price fluctuations, at the country level. To the extent that the law of one price holds for commodities, and if changes in commodity prices have differential effects on the domestic cost of any two countries, commodity price changes will affect the real exchange rate between those two countries. As we show when we describe the model in Section 1, this is trivially true in theory. The relevant question is how important this is quantitatively. This motivates the empirical analysis we describe in Section 2.

2 The Model

We consider a world with a finite number of countries, each one inhabited by a representative consumer. In each country, there are different varieties of labor, intermediate goods, and commodities. In particular we assume that there are

\[ j = 1, 2, \ldots, J \] types of labor

\[ i = 1, 2, \ldots, N \] types of intermediate goods

\[ h = 1, 2, \ldots, H \] types of commodities

where \( J \geq N \). We assume that there is at least the same number of labor varieties as intermediate goods. If this is the case, all varieties will be produced in each country. If not, some varieties (the ones with lowest values of TFP) will not be produced in some countries. This simplifies some of the algebra below. We also assume that for each commodity, there is a fixed factor that is used in production.

All technologies will be assumed Cobb-Douglas in all countries. Countries will differ in their

\(^1\)Technically, this is only a necessary, but not a sufficient condition. What we need is that the cone for each country is such that all varieties are produced in the countries we will be analyzing.

\(^2\)We explain below how this matters and what happens if this is not the case.
endowments and in their production function parameters, including total factor productivity associated to each production function.

The fixed factors used in the production of commodities and the different varieties of labor are non-traded. But intermediate goods and commodities are traded goods.

In what follows, we describe in detail preferences and the production structure of one of the economies. To fix ideas, consider the economy whose currency is used for international transactions - the United States in our empirical application. We now describe the environment in this economy, without any specific supra-index. Those will be introduced afterwards, once we consider more than a single country.

Preferences are given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t), \]

where \( C_t \) represents consumption of a non-traded final good. This final consumption good should be seen as an aggregate of a very large number of different varieties. We assume these varieties are non-traded to make the model consistent with the overwhelming evidence of lack of the law-of-one price in final goods. In this sense, the model below adopts the view of Burstein et. el (2001), who argue that an important share of final good prices have a non-traded component.

There is also a cash-in-advance constrained of the form

\[ P_tC_t \leq M_t. \]

Preferences can vary across countries, though they will be irrelevant in the discussion that follows. All we exploit from the model comes from its production structure.

Production of all goods (final, intermediate, and commodities) involves inputs of all types of labor. Labor for the production of each of them is aggregated from all varieties using Cobb-Douglas production functions. The total labor endowment of each varieties is equal to \( e_j \), which
can be country specific.

The final good is produced according to the technology

\[ C_t = Y_t = Z^y_t \left( \prod_{j=1}^{J} [n^y_t(j)]^{\psi^y(j)} \right)^{\alpha} (q_t)^{1-\alpha} \]

where \(0 < \alpha < 1\) and \(\sum_{j=1}^{J} \psi^y(j) = 1\). We let \(\alpha\) to be country specific. The input \(q_t\) is a domestic, non-traded good produced using a finite number of traded intermediate goods according to the production function

\[ q_t = \prod_{i=1}^{N} q_t(i)^{\varphi(i)} \]

where \(\sum_{i=1}^{N} \varphi(i) = 1\). Again, the \(\varphi(i)'s\) can be country specific.

In each country, each variety of intermediate good \(i\) is produced using labor and a finite number of commodities \(x_t(h)\), where \(h = 1, 2, ..., H\). The production function is

\[ Q_t(i) = z_t(i) \left( \prod_{j=1}^{J} [n^Q_t(j)]^{\psi^Q(i,j)} \right)^{\beta} \left( \prod_{h=1}^{N} [x_t(h)]^{\gamma(i,h)} \right)^{1-\beta} \]

where \(\sum_{h=1}^{N} \gamma(i, h) = 1\) and \(\sum_{j=1}^{J} \psi^Q(i, j) = 1\) for all \(i\), and \(0 < \beta < 1\). We also let \(\beta\), \(\psi^Q(j)\) and \(\gamma(i, h)\) to be country specific.

Finally, in each country there is a technology to produce the commodities given by

\[ X^h_t = Z^h_t \left( \prod_{j=1}^{J} [n^h_t(j)]^{\psi^h(j)} \right)^{\eta_h} E_t(h)^{1-\eta_h}, \text{ for all } h \]

where we can also let \(\eta_h, \psi^h(j)\) be country specific. Here, \(E_t(h)\) is the amount of the endowment that is used in the production of the commodity. As the endowment is not traded, as long as \(E_t(h) > 0\), a positive amount of the commodity will be produced. Naturally, if \(E_t(h) = 0\) for a
particular country, production of that commodity will be zero and, as long as some is used in the production of intermediate goods, it will be imported.

2.1 Prices

Since there is perfect competition, prices will be equal to marginal costs. With Cobb-Douglas production functions, marginal costs are Cobb-Douglas functions of factor prices. Thus, the price level in the numeraire country will be

\[ P_t = \frac{\kappa^y}{Z^y_t} \left( \prod_{j=1}^{J} [W_t(j)]^{\psi_y(j)} \right)^{\alpha} (P^q_t)^{1-\alpha}, \tag{1} \]

where \( \kappa^y \) is a constant that depends on the exponents in the Cobb-Douglas.

Similarly,

\[ P^q_t = \kappa^q \prod_{i=1}^{N} P^q_t(i)^{\psi(i)}. \tag{2} \]

The prices of the intermediate goods are

\[ P^q_t(i) = \frac{\kappa^q(i)}{z_t(i)} \left( \prod_{j=1}^{J} [W_t(j)]^{\psi_Q(i,j)} \right)^{\beta} [\prod_{h=1}^{N} P_t(h)^{\gamma(i,h)}]^{1-\beta}. \tag{3} \]

where again, \( \kappa^q \) and \( \kappa^q(i) \) depend on parameters of the production functions. Finally, the prices of the commodities are

\[ P_t(h) = \frac{\kappa^h}{Z^h_t} \left( \prod_{j=1}^{J} [W_t(j)]^{\psi_h(j)} \right)^{\eta_h} P^E_t(h)^{1-\eta_h}, \text{ for all } h, \]

where \( P^E_t(h) \) is the price of the fixed factor.
Taking natural logarithms in (1), (2) and (3) delivers

\[
\ln P_t = \ln \frac{\kappa^y}{Z_t^y} + \alpha \left( \sum_{j=1}^{J} \psi^y(j) \ln W_t(j) \right) + (1 - \alpha) \ln P_t^q
\]

\[
\ln P_t^q = \ln \kappa^q + \sum_{i=1}^{N} \varphi(i) \ln P_t^q(i)
\]

\[
\ln P_t^q(i) = \ln \frac{\kappa^q(i)}{z_t(i)} + \beta \left[ \sum_{j=1}^{J} \psi^q(i, j) \ln W_t(j) \right] + (1 - \beta) \left[ \sum_{h=1}^{N} \gamma(i, h) \ln P(h)_t \right].
\]

From these expressions it follows that

\[
\ln P_t = \ln \frac{\kappa^y}{Z_t^y} + \alpha \left( \sum_{j=1}^{J} \psi^y(j) \ln W_t(j) \right)
\]

\[
+ \left( 1 - \alpha \right) \left[ \ln \kappa^q + \sum_{i=1}^{N} \varphi(i) \left( \ln \frac{\kappa^q(i)}{z_t(i)} + \beta \left[ \sum_{j=1}^{J} \psi^q(i, j) \ln W_t(j) \right] + (1 - \beta) \left[ \sum_{h=1}^{N} \gamma(i, h) \ln P(h)_t \right] \right) \right]
\]

or

\[
\ln P_t = \ln \kappa^y + (1 - \alpha) \left[ \ln \kappa^q + \sum_{i=1}^{N} \varphi^q(i) \ln \kappa^q(i) \right] - \left( \ln Z_t^y + (1 - \alpha) \sum_{i=1}^{N} \varphi(i) \ln z_t(i) \right)
\]

\[
+ \sum_{j=1}^{J} \ln W_t(j) \left[ \alpha \psi^y(j) + (1 - \alpha) \beta \sum_{i=1}^{N} \varphi(i) \psi^q(i, j) \right]
\]

\[
+ (1 - \alpha)(1 - \beta) \sum_{h=1}^{N} \ln P(h)_t \sum_{i=1}^{N} \varphi(i) \gamma(i, h).
\]

Summarizing, the log of the aggregate price level will be a log-linear function of some constants, productivity shocks in final and intermediate goods on one hand, and prices (of commodities and labor) on the other. Note that weights on all prices are non-negative, since they
are products of exponents in the production functions. Note also that they add up to one, since

\[
\sum_{j=1}^{J} \left[ \alpha \psi^y(j) + (1 - \alpha) \beta \sum_{i=1}^{N} \varphi(i) \psi^Q(i, j) \right] + (1 - \alpha)(1 - \beta) \sum_{h=1}^{N} \sum_{i=1}^{N} \varphi(i) \gamma(i, h)
\]

\[
= [\alpha + (1 - \alpha) \beta \sum_{i=1}^{N} \varphi(i)] + (1 - \alpha)(1 - \beta) \sum_{i=1}^{N} \varphi(i)
\]

\[
= \alpha + (1 - \alpha) \beta + (1 - \alpha)(1 - \beta) = 1
\]

As long as some commodity is produced in this economy, cost minimization in that industry implies that

\[
W_t(\tilde{j}) = P(h_t) \eta_h \psi^h(\tilde{j}) Z_t^h \left( \prod_{j=1, j \neq \tilde{j}}^{J} \left[ \frac{n^h_t(j)}{n^h_t(\tilde{j})} \right] \psi^h(j) \right)^{\eta_h - 1} \left( \frac{E_t(h)}{n^h_t(\tilde{j})} \right)^{1 - \eta_h}
\]

for all \( \tilde{j} \).

Therefore,

\[
\ln W_t(\tilde{j}) = \ln P(h_t) + \ln \eta_h \psi^h(\tilde{j}) Z_t^h + (\eta_h - 1) \sum_{j=1, j \neq \tilde{j}} \psi^h(j) \ln \left[ \frac{n^h_t(j)}{n^h_t(\tilde{j})} \right] + (1 - \eta_h) \ln \left( \frac{E_t(h)}{n^h_t(\tilde{j})} \right)
\]

which means that we can replace the log-wages on equation (4) above by a log-linear expression on commodity prices, productivity shocks in the commodities sector and quantities (how different labor varieties are allocated in the production of commodities).

Clearly, (5) can be used to eliminate wages in (4) and leave the price level as a log-linear function of constants, productivity shocks in all sectors and commodity prices, where the sum of the coefficients in all commodity prices is equal to 1. By imposing the law of one price on commodities, we can use that expression (and a similar one for a different country) to derive a relationship between the real exchange rate and the commodity prices. The expression, however, is ugly and hard to interpret, so we now consider a simplified version, where these changes can
be seen clearly.

But before that, we want to discuss two caveats. The first is that one could also express the price level as a log-linear function of the prices of intermediate goods. So why not stopping there? The reason to replace one more layer in the input output matrix is that the universe of commodities is very small, while the universe of intermediate goods is enormous. This notable simplifies the empirical analysis. But in theory, there is no reason why not stopping at the intermediate good level, given our assumption that intermediate goods are also traded. The second is that we used equation (5) for a particular commodity, but not for other. Are there more than one way to express costs of domestic final goods in terms of commodities? Not really, since the quantities that show up in equation (5) do depend on the way the other commodity prices move. Indeed, those quantities are correlated to commodity prices, as well as to productivity shocks. Thus, commodity prices move local costs through the direct effect identified in equations (4) and (5) and through the indirect effect they have on the allocation of resources.

2.2 A simple case

Assume single labor type and let the production function be

\[ C_t = Y_t = Z_t^y(n_t)^\alpha (q_t)^{1-\alpha} \]

where \( q_t \) is a domestic, traded intermediate good that can be produced according to

\[ Q_t = z_t[n_t^q]^{\beta_n}[x_t]^{\beta_x}[m_t]^{\beta_m}. \]

---

If one is more willing to assume the law of one price holds for commodities than for intermediate goods, then reducing costs to commodity prices has an additional advantage.
Assume in the numeraire country there is a technology to produce the commodity \( x_t \),

\[
X_t = Z_t^h [n_t^x] \eta E^{1-\eta}
\]

where for simplicity we assume the endowment \( E \) is fixed. The commodity \( m_t \) must be imported.

From the cost minimization conditions of the commodity sector, we have

\[
W_t = P_t^x \eta Z_t^h \left[ \frac{E}{n_t^x} \right]^{1-\eta}
\]

so we can use this condition to express the wage as a function of commodity prices plus factor reallocations in the final equation.

The marginal cost of the final good can be expressed as

\[
P_t = \frac{\kappa^y}{Z_t^y} (W_t)^\alpha (P_t^q)^{1-\alpha}.
\]

But

\[
P_t^q = \frac{\kappa^q}{z_t} [W_t]^{\beta_n} [P_t^x]^{\beta_x} [P_t^m]^{\beta_m},
\]

so

\[
P_t = \frac{\kappa^y}{Z_t^y} (W_t)^\alpha \left( \frac{\kappa^q}{z_t} [W_t]^{\beta_n} [P_t^x]^{\beta_x} [P_t^m]^{\beta_m} \right)^{1-\alpha}
\]

Taking logs,

\[
\ln P_t = \ln \left( \frac{\kappa^y}{Z_t^y} \right) + (\alpha + (1 - \alpha) \beta_n) \ln W_t + (1 - \alpha) \left( \ln \left( \frac{\kappa^q}{z_t} \right) + \beta_x \ln P_t^x + \beta_m \ln P_t^m \right)
\]

but using the cost minimization condition derived above gives

\[
\ln P_t = \ln \left( \frac{\kappa^y}{Z_t^y} \right) + (\alpha + (1 - \alpha) \beta_n) \left[ \ln P_t^x + \ln \eta Z_t^h + (1 - \eta) \ln \left[ \frac{E}{n_t^x} \right] \right] + (1 - \alpha) \left( \ln \left( \frac{\kappa^q}{z_t} \right) + \beta_x \ln P_t^x + \beta_m \ln P_t^m \right)
\]
ln \( P_t \) = \( \ln \frac{\kappa^y}{Z_t^y} + (1 - \alpha) \ln \frac{\kappa^q}{z_t} + (\alpha + (1 - \alpha)\beta_n) \ln \eta Z_t^h + (\alpha + (1 - \alpha)\beta_n)(1 - \eta) \ln \left[ \frac{E}{n_t^x} \right] \\
+ (1 - \alpha)\beta_m \ln P_t^m + (\alpha + (1 - \alpha)\beta_n + (1 - \alpha)\beta_x) \ln P_t^x \\

Note that \( (1 - \alpha)\beta_m + \alpha + (1 - \alpha)\beta_n + (1 - \alpha)\beta_x = 1 \), so once we use the law of one price for the two commodity prices, we get the nominal exchange rate on the right hand side and therefore a nice expression for the real exchange rate.

In this case, we have on the right hand side commodity prices, productivity shocks and factor allocations.

### 2.3 The real exchange rate

In general, then, we can write the solution for the price level in the US as

\[
\ln P_{US}^t = k - Z_{US}^t + \sum_{h=1}^{N} a_{US}^h \ln P_{US}^t(h)
\]

where \( Z_{US}^t \) is a combination of productivity shocks and factor allocations, and where

\[
\sum_{h=1}^{N} a_{US}^h = 1,
\]

so we can use any one of those commodity prices and write all other relative to that one. Similarly, for a different country we have

\[
\ln \tilde{P}_t = \tilde{k} - \tilde{Z}_t + \sum_{h=1}^{N-1} \tilde{a}_h \ln \tilde{P}_t(h)
\]

where prices are now relative to the one chosen as numeraire.
Since commodities are traded, the law of one price implies

\[
P_t^{US}(h)\tilde{S}_t = \tilde{P}_t(h),
\]

where \( \tilde{S}_t \) is the nominal exchange rate between the foreign country and the benchmark country.

Then, we can write

\[
\ln \tilde{P}_t = \tilde{k} - \tilde{Z}_t + \sum_{h=1}^{N-1} \tilde{a}_h (\ln P_t^{US}(h) + \ln \tilde{S}_t)
\]

or

\[
\ln \tilde{P}_t - \ln S_t = \tilde{k} - \tilde{Z}_t + \sum_{h=1}^{N-1} \tilde{a}_h \ln P_t^*(h)
\]

Taking the difference between the log prices in both countries, we thus obtain

\[
\ln P_t^* - \left( \ln \tilde{P}_t - \ln \tilde{S}_t \right) = k - Z_t - \left( \tilde{k} - \tilde{Z}_t \right) + \sum_{h=1}^{N-1} \tilde{a}_h^U \ln P_t^*(h) - \sum_{h=1}^{N-1} \tilde{a}_h \ln P_t^*(h)
\]

or

\[
\ln \frac{P_t^* S_t}{\tilde{P}_t} = \left( k - \tilde{k} \right) + \sum_{h=1}^{N-1} (\tilde{a}_h^U - \tilde{a}_h) \ln P_t^*(h) + \left( \tilde{Z}_t - Z_t \right).
\]

This regression equation is used in the empirical section.

Note that commodity prices will affect the RER as long as \((a_h^U - \tilde{a}_h) \neq 0\). This may be the case for one of two reasons. First, it could be that the parameters of the Cobb-Douglas production functions differ across countries. Second, it could be that the endowment of commodities differ across countries such that some commodities are produced in some countries but not in others. Clearly, one should expect the stochastic disturbance \((\tilde{Z}_t - Z_t)\) to be correlated with the commodity prices, so the estimates obtained would not be consistent estimators of the parameters \((a_h^U - \tilde{a}_h)\). However, for the discussion of how much of the variance of RER can we account with a few commodity prices, which is what we do in this paper, the correlation is
irrelevant.

To match quantitatively the parameters $(\delta_h^U - \tilde{\delta}_h)$, one would need very detailed information on the output-input matrices of the two countries involved, information we do not have.\footnote{The algebra does however indicate that the commodities that are produce in the two countries should have a stronger weight - as it is clearly the case in the small open economy literature in examples as Chile with copper and Norway with oil. We are currently working on studying empirically those implications.} Thus, in our empirical section we propose a different strategy.

3 Empirical results

We use equation (6) for the empirical analysis. We first selected the 34 most traded commodities and picked the 10 commodities with the highest share of world trade value in 2012, so we will be working with 9 relative prices (we normalize prices by the price of Wheat). In Table 1 we list the 34 commodities ranked according to their shares in the value of world trade, together with the classification used to define each commodity. Our selection of 10 commodities include: Petroleum, Natural Gas, Gold, Coal, Iron, Soybeans, Copper, Wheat, Wood, and Beef.\footnote{We do not include Fish, since we could not find a price series that cover the entire period (1960-2014). The price series of Fish in the World Bank Commodity Price Data begins only in 1979. Besides that, the classification Fish can also be considered too broad when compared to meat, for example, where there is the distinction between Beef and Pork.} In Appendix 1 we explain the data. All together, the commodities in Table 1 accounted for 20% of total world trade value in 2012, again, without taking into account the share they account for in the cost of intermediate goods. We then run regressions of real exchange rates on relative commodity prices, treating productivity shocks and the allocation of resources as unobservables. We consider the bilateral real exchange rates between the US and Germany (Mark until 2000, EURO since then), Japan, and the UK. We use monthly data over the period 1960:M1 - 2014:M11.

First, we run a regression of the RER on the 9 relative price series and selected the commodities whose coefficients where statistically significant at a 10% level. We then run the regression again until we had only statistically significant coefficients. The results are depicted in Table 2.
### Table 1: Commodity List

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Share of world trade in 2012</th>
<th>SITC (rev.4)</th>
<th>Commodity</th>
<th>Share of world trade in 2012</th>
<th>SITC (rev.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Petroleum (crude)</td>
<td>11.28%</td>
<td>033</td>
<td>(19) Platinum</td>
<td>0.17%</td>
<td>681.2</td>
</tr>
<tr>
<td>(2) Natural Gas</td>
<td>1.44%</td>
<td>343</td>
<td>(20) Pork</td>
<td>0.17%</td>
<td>012.2</td>
</tr>
<tr>
<td>(3) Gold</td>
<td>1.40%</td>
<td>971.01</td>
<td>(21) Coffee</td>
<td>0.13%</td>
<td>071.1</td>
</tr>
<tr>
<td>(4) Coal</td>
<td>0.69%</td>
<td>321</td>
<td>(22) Rice</td>
<td>0.13%</td>
<td>421-423</td>
</tr>
<tr>
<td>(5) Iron</td>
<td>0.67%</td>
<td>281</td>
<td>(23) Cotton</td>
<td>0.11%</td>
<td>026.3</td>
</tr>
<tr>
<td>(6) Fish</td>
<td>0.31%</td>
<td>034</td>
<td>(24) Rapeseed</td>
<td>0.07%</td>
<td>222.6</td>
</tr>
<tr>
<td>(7) Soybeans</td>
<td>0.29%</td>
<td>222.2</td>
<td>(25) Tobacco</td>
<td>0.06%</td>
<td>121</td>
</tr>
<tr>
<td>(8) Copper</td>
<td>0.27%</td>
<td>283.1</td>
<td>(26) Cocoa</td>
<td>0.05%</td>
<td>072.1</td>
</tr>
<tr>
<td>(9) Wheat</td>
<td>0.27%</td>
<td>041</td>
<td>(27) Milk</td>
<td>0.05%</td>
<td>022.1</td>
</tr>
<tr>
<td>(10) Wood</td>
<td>0.26%</td>
<td>247, 248</td>
<td>(28) Nickel</td>
<td>0.05%</td>
<td>284</td>
</tr>
<tr>
<td>(11) Beef</td>
<td>0.22%</td>
<td>011</td>
<td>(29) Banana</td>
<td>0.04%</td>
<td>057.12</td>
</tr>
<tr>
<td>(12) Palm Oil</td>
<td>0.21%</td>
<td>422.2</td>
<td>(30) Ethanol</td>
<td>0.04%</td>
<td>512.15</td>
</tr>
<tr>
<td>(13) Corn (maize)</td>
<td>0.20%</td>
<td>044</td>
<td>(31) Wool</td>
<td>0.04%</td>
<td>268</td>
</tr>
<tr>
<td>(14) Rubber</td>
<td>0.19%</td>
<td>231</td>
<td>(32) Olive Oil</td>
<td>0.03%</td>
<td>421.4</td>
</tr>
<tr>
<td>(15) Silver</td>
<td>0.19%</td>
<td>681.1</td>
<td>(33) Tea</td>
<td>0.03%</td>
<td>074.1</td>
</tr>
<tr>
<td>(16) Wine</td>
<td>0.19%</td>
<td>112.1</td>
<td>(34) Orange</td>
<td>0.03%</td>
<td>057.11</td>
</tr>
<tr>
<td>(17) Sugar</td>
<td>0.18%</td>
<td>026.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** SITC (rev4) stands for Standard International Trade Classification (revision 4).

**Source:** Comtrade

In the case of Germany eight commodity prices are statistically significant at the 1% level and the $R^2$ is 0.53. In the case of UK, all commodities are statistically significant, but we can also see that it is the regression with the lowest $R^2$, equal to 0.53. Finally, in the case of Japan, only six commodities are statistically significant, but the regression has the highest $R^2$, equal to 0.84. In Figures 1.a to 1.c, we show the RER for each of these two countries, together with the value of the RER generated by the regressions showed in Table 2. The match is remarkably good.
Table 2: Real Exchange Rate and 9 Relative Commodity Prices (1960:M1 - 2014:M11)

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th></th>
<th>UK</th>
<th></th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beef</td>
<td>0.261***</td>
<td>Beef</td>
<td>0.084***</td>
<td>Beef</td>
<td>0.302***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Coal</td>
<td></td>
<td>Coal</td>
<td>-0.111***</td>
<td>Coal</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>-0.208***</td>
<td>Copper</td>
<td>-0.099***</td>
<td>Copper</td>
<td>-0.106***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Gold</td>
<td>-0.182***</td>
<td>Gold</td>
<td>-0.068***</td>
<td>Gold</td>
<td>-0.455***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Iron</td>
<td>0.289***</td>
<td>Iron</td>
<td>0.197***</td>
<td>Iron</td>
<td>0.442***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>0.130***</td>
<td>Natural Gas</td>
<td>0.046***</td>
<td>Natural Gas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petroleum</td>
<td>-0.125***</td>
<td>Petroleum</td>
<td>-0.072***</td>
<td>Petroleum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soybeans</td>
<td>-0.129***</td>
<td>Soybeans</td>
<td>0.068***</td>
<td>Soybeans</td>
<td>-0.088***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.002)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Wood</td>
<td>-0.098***</td>
<td>Wood</td>
<td>-0.076***</td>
<td>Wood</td>
<td>-0.395***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.53</td>
<td>$R^2$</td>
<td>0.43</td>
<td>$R^2$</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Note: p-values in parentheses.
in all three cases.

Second, we divide the sample into three sub periods (1960:M01 - 1978:M12, 1979:M01 - 1997:M12, 1998:M01 - 2014:M11) and repeat the exercise described above in each of the sub periods. We show the $R^2$'s of the regressions in Table 3 and confirm that the previous results still hold when we analyze the sub periods separately.
Table 3: $R^2$ - sub periods

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.53</td>
<td>0.94</td>
<td>0.65</td>
<td>0.70</td>
</tr>
<tr>
<td>UK</td>
<td>0.43</td>
<td>0.57</td>
<td>0.60</td>
<td>0.52</td>
</tr>
<tr>
<td>Japan</td>
<td>0.84</td>
<td>0.96</td>
<td>0.89</td>
<td>0.50</td>
</tr>
</tbody>
</table>

In the previous exercises we chose the commodities that deliver a good fit of the RER. One could argue that, since the list of commodity prices is long, it could be just luck that some of those series have a statistical relationship with the RER. To further explore this issue, we run regressions for each decade, select the commodities whose coefficients are statistically significant in that decade, and then run regressions using these commodities for the subsequent decade. As long as the input-output matrix and the production structures of the two involved economies do not change much over the following decade,\footnote{This may be hard to believe in the three decades that had been characterized as the "globalization" age, with China and other emerging economies dramatically transforming world trade. This is one reason why we do this exercise only one decade at a time.} those regressors should do a good job in explaining the real exchange rate in the following decade. The results of this exercise are shown in Table 4. For example, in column 1970-1979, in the case of Germany, we run the regression for the period 1960-1969, pick the commodities that are statistically significant, and then we run the regression for the period 1970-1979 using this group of commodities (testing out-of-sample fit).

One potential problem with the previous exercises is that commodity prices are highly correlated. Thus, because of collinearity, t-statistics depend on the inclusion of the other regressors, that are then eliminated in the new regression. Furthermore, the high correlation among commodity prices suggests that there may be a few common factors that drive most of the variation.
Table 4: $R^2$ - regressors from previous decade

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.93</td>
<td>0.87</td>
<td>0.48</td>
<td>0.43</td>
</tr>
<tr>
<td>UK</td>
<td>0.83</td>
<td>0.50</td>
<td>0.65</td>
<td>0.56</td>
</tr>
<tr>
<td>Japan</td>
<td>0.93</td>
<td>0.88</td>
<td>0.62</td>
<td>0.60</td>
</tr>
</tbody>
</table>

in commodity prices. In particular, if we let $f_t$ denote a $r \times 1$ vector of common factors, the underlying assumption is that commodity prices have a factor structure of the form

$$\ln P_t(h) = b_h + c_h' f_t + u_{ht} \text{ for } h = 1, 2, ..., H,$$

(7)

where $c_h$ is an $r \times 1$ vector of factor loadings, and $u_{ht}$ is a residual orthogonal to the factors $f_t$. The crucial point of this specification is that the number of factor $r$ is smaller than the number of commodity prices $H$.

There are several methods to estimate and identify unobserved factors and factor loadings. We use what Stock and Watson (2011) call “second generation dynamic factor models” whereby the factors are estimated using cross-sectional averages with possibly different weights. Roughly speaking, the idea is to compute, for each time period, a weighted cross-sectional average of commodity prices to obtain the space spanned by the factors. We use the methodology of principal components to estimate both the unobserved factors $f_t$ and the factor loadings $c_h$. The methodology produces a set of orthogonal factors (as many as the number of commodities) which can be ordered according to their contribution to the overall variance of the set of commodity prices. Using a set of 9 monthly relative commodity prices for the period 1960:M1 through
2014:M12, we found that just the first two factors explain about 80% of the variance of the series. Four factors explain about 92% of the variance. Appendix 2 discusses some details of the implementation of the factor model.

Once we have estimated the orthogonal factors, we run the regressions 6 using the six unobserved factors that are most important in explaining the overall variance of commodity prices. The results are reported in Table 5.

Table 5: $R^2$ - 6 Factors

<table>
<thead>
<tr>
<th></th>
<th>1960-2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.35</td>
</tr>
<tr>
<td>UK</td>
<td>0.31</td>
</tr>
<tr>
<td>Japan</td>
<td>0.76</td>
</tr>
</tbody>
</table>

With six factors, we can explain over 75% of the variance of the RER with respect to Japan, a noteworthy result. In contrast, these factors explain about 1/3 of the variance of the RER with respect to Germany and the UK, still a respectable amount. It is interesting to note, on the other hand, that the RER with respect to Japan is the most less volatile of the three, as may become evident by inspecting Figures 1.a through 1.c.

4 Concluding comments

In this paper we argue that commodity price fluctuations are an important source of volatility in real exchange rates among developed economies. In particular, we study the bilateral exchange rates between the United States, Japan, Germany and the United Kingdom.
Using a standard model that explicitly accounts for commodities, we derive a relationship between real exchange rates on one hand and productivity shocks and commodity prices on the other. Productivity shocks cannot be observed, but they can and have been widely estimated using the Solow residual methodology. All measures available imply a volatility of productivity shocks that is orders of magnitude lower than the one observed in commodity price shocks. One could be naturally lead to think therefore that most of the volatility of RER should be accounted by the more volatile component. In addition, one would expect that periods - say decades - with lower commodity price volatility should therefore lead to periods with lower RER volatility. Is it the case that the post Bretton-Woods period lead to more RER volatility because of the prevailing free floating era or just because the turmoil that followed the oil price shock of the seventies lead to higher commodity price volatility? Does it matter at which frequency do we measure the volatility? So far, we have presented the empirical analysis done treating the data as it is. We are extending our data set to incorporate the period 1950 to 1960 and extending the analysis using different frequencies (monthly versus quarterly or yearly observations).
References


A Appendix 1: Data

Regarding the nominal exchange rates, we used the official monthly series (end of period). For the price indexes, we used the monthly Consumer Price Index (CPI) of each country. Finally, here we list the data source for the price series of each commodity.

(1) Beef - Source: Global Financial Data, Ticker: CMWBEEFM
(2) Coal - Source: Global Financial Data, Ticker: CMCOALM
(3) Copper - Source: Global Financial Data, Ticker: CY NYD
(4) Gold - Source: Global Financial Data, Ticker: XAU BD
(5) Iron - Source: Global Financial Data, Ticker: CMIRONOM
(6) Natural Gas - Source: World Bank Commodity Price Data (Pink Sheet), Natural Gas Index
(7) Petroleum - Source: Global Financial Data, Ticker: BRT D
(8) Soybeans - Source: Global Financial Data, Ticker: SYB TD
(9) Wheat - Source: Global Financial Data, Ticker: W USSD
(10) Wood - Source: World Bank Commodity Price Data (Pink Sheet), Timber Index

B Appendix 2: Factor model and principal components

We estimate the factor model using monthly data on the 9 commodity relative prices. We assume that the (log) relative prices of the commodity $h = 1, 2, ..., 9$ can be well represented by a factor model of the form

$$p_t(h) = b_h + c_h f_t + u_{ht} \text{ for } h = 1, 2, ..., H$$

where $f_t$ is an $r \times 1$ vector of unobserved factor with $r << H$, $c_h$ is an $r \times 1$ vector of factor loadings, and $u_{ht}$ is a residual orthogonal to $f_t$ at all leads and lags.

The methodology used to estimate the space spanned by the factors is to compute cross-sectional averages of the commodity prices. To implement the methodology, it is convenient to work with normalized data. For that reason let $\bar{P}_t$ denote the stacked vector of standardized commodity prices at time $t$ (demeaned and with unit variance). Then we can write the factor model as

$$\bar{P}_t = \Lambda F_t + u_t,$$

---

Stock and Watson (2011) discuss the necessary conditions to consistently estimate the space spanned by the factors.
where $\Lambda$ is a matrix with the factor loadings in its rows, $F_t$ is a vector with the factors associated with the standardized data, and $u_t$ is a vector with the residuals.

As it is well known, it is not possible to separately identify the factors $F_t$ and the factor loadings $\Lambda$. Indeed, given an arbitrary invertible $r \times r$ matrix $\Omega$, the previous model is observationally equivalent to a model of the form $\tilde{P}_t = \tilde{\Lambda} \tilde{F}_t + u_t$, where $\tilde{\Lambda} = \Lambda \Omega^{-1}$ and $\tilde{F}_t = \Omega F_t$.

One simple way to form identified factor models is using principal components. We construct the estimator of $F_t$ as a weighted average of $\tilde{P}_t$ using an $H \times r$ matrix $W$ of weights normalized so that $W'W/H = I$ such that $\hat{F}_t = H^{-1}W'\tilde{P}_t$. The principal component estimator of $F_t$ uses $W = \Lambda$, where $\Lambda$ is the matrix of eigenvectors of the sample covariance matrix of $\tilde{P}_t$ associated its $r$ largest eigenvalues. The following table reports the percentage of the variance of the 9 relative commodity prices explained by the factors.

<table>
<thead>
<tr>
<th>Component #</th>
<th>Contribution</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>67.0</td>
<td>67.0</td>
</tr>
<tr>
<td>2</td>
<td>13.1</td>
<td>80.1</td>
</tr>
<tr>
<td>3</td>
<td>6.1</td>
<td>86.2</td>
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<td>4</td>
<td>5.7</td>
<td>91.9</td>
</tr>
<tr>
<td>5</td>
<td>3.1</td>
<td>95.0</td>
</tr>
<tr>
<td>6</td>
<td>3.0</td>
<td>98.0</td>
</tr>
<tr>
<td>7</td>
<td>1.1</td>
<td>99.1</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>99.7</td>
</tr>
<tr>
<td>9</td>
<td>0.3</td>
<td>100.0</td>
</tr>
</tbody>
</table>

The next three figures show the in-sample goodness of fit of the model (that is, $\hat{P}_t(h) = \Lambda^h \hat{F}_t$) for four arbitrary relative commodity prices (oil/wheat, copper/wheat, soybeans/wheat, and gold/wheat) using just one factor ($r = 1$), two factors ($r = 2$) and four factors ($r = 4$). As can be observed in the figures, even the two factor model does a good job tracking the evolution of the proposed relative commodity prices.
Figure 2: Data and fit of model with just one factor
Figure 3: Data and fit of model with two factors
Figure 4: Data and fit of model with four factors