Abstract

This article studies the cyclical behavior of asset prices and market beliefs. The model analyzes dynamic rational expectations equilibrium in line with Grossman & Stiglitz (1980). I examine the equilibrium in the asset market with a continuum of risk averse investors who can acquire private costly information in the information market. The information discovery displays increasing returns to scale. A representative investor plays a mixed strategy between acquiring information and not. The identity of the marginal trader fluctuates over the business cycle. The proportion of informed and uninformed investors fluctuates with the cycle. The proportion of informed investors increases in booms and retracts during recessions. Uncertainty measured by the conditional variance of uninformed investors increases in recessions. The model amplifies the excess return paid over the risk-free rate, introducing an alternative explanation for the equity risk premium.
1 Introduction

Economists seem to have reached a consensus about the counter-cyclical behavior of uncertainty\(^1\). However, there is much less agreement on what are the drivers of uncertainty during these periods. Even more so, the question whether uncertainty causes recessions (or vice-versa) remains open.

This paper studies the information markets, as the main mechanism that generates the counter-cyclical behavior of uncertainty.

The novel aspect of the paper is the interaction between the signalling problem of firms when considering whether to disclose information, and the incentives for traders to acquire private information which relatively increase the precision of the posterior belief. The model amplifies the excess return paid over the risk-free rate, suggesting an alternative explanation for the equity risk premium.

While many models on the consequences of a hike in uncertainty have proven to be very promising, there is not a good explanation for the counter-cyclical behavior of the uncertainty. To tackle this question, it is important to capture the incentives of the agents to acquire or produce private source of information, but also to consider the strategic disclosure decision of privileged informed agents, such as firms’ managers, governments, and monetary authorities. This paper shows that voluntary disclosures by managers, along with the fluctuating incentives to produce financial information can help explain the counter-cyclical behavior of uncertainty, as well as can help understand its consequences on the asset prices.

The first contribution of the paper is to show that neither voluntary disclosures nor private discovery of information (the information acquisition decision) are, by its own, sufficient to increase the agents’ dispersion of beliefs, which represents a measure of uncertainty in this model, in a counter-cyclical fashion. In line with Acharya, DeMarzo, and Kremer [2011] and Shin [2003], the manager, self-interested acting, carefully weights the consequences of the voluntary disclosure on the stock price of the firm, which he seeks to maximize. With a certain probability, the manager has

\(^1\)Bachmann, Elstner, and Sims [2013], Bloom [2009], Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry [2012] and Johnson [2004], only to mention a few of the recent works, show data on several measures of uncertainty, that drastically deteriorate during recession
more information about the true distribution of liquidation value of the firm. This event remains private to the manager, with some probability, giving him the ability to conceal information by mimicking the uninformed type of manager. The paper studies the interaction of the manager and sophisticated traders, that rationally learn from observing the market price in the spirit of Grossman and Stiglitz [1980]. Disclosures are verifiable, and directed to the information producers, which in turn improves their ability to analyse and communicate their discoveries about the liquidation value of the firm, and therefore increase the precision of the signal provided to the traders.

The model highlights the importance of the interaction of the two driving channels in determining the traders’ incentives to acquire information. On one side, the acquisition of the private signal improves the precision of the posterior of the trader. On the other side, since the realization of the private signal is partially reveal through the market price, a highly precise private signal also improves precision of the public signal. Therefore the incentives to acquire the signal might decline, depending on the dynamics of the price for the financial information.

Abstracting from the information acquisition problem, and only considering a model in which disclosures improve the precision of the traders’ prior about the liquidation value of the firm, fails to generate any fluctuation in the dispersion of beliefs. Then, this model does not increase the uncertainty as measured in this paper. For that reason, it is crucial to analyze a broader framework, in which traders are able to choose their type (informed or uninformed). In such a framework, upon a disclosure of the manager, a higher precision of the private signal increases the traders’ incentives to acquire information in the market for financial information. However, again without any further structure, the higher precision of the private signals also improve the precision of the public signal embedded in the stock price of the firm. In fact, as long as the information discovery displays a constant return to scale, the dispersion of the traders’ beliefs also remain constant - irrespectively of the relative precision of the public and private signals.

The second contribution is to show that an increasing return in the information discovery function, can create the incentives such that the proportion of informed traders to relatively increase upon a disclosure from the manager. Increasing returns in the market for financial information strengthens the strategic complementarity feature of private information. In particular, a larger
demand for information reduces the price for private signal in such a way that, although the public signal improves its precision, it is optimal in the aggregate to acquire more private signals. The current framework is the simplest possible that can deliver a counter-cyclical behavior of the cross-sectional dispersion of beliefs without assuming an exogenous shock that increases the relative uncertainty of the traders.

The attention on the paper is drawn to the asset pricing implication of the manager’s disclosures, and the information acquisition by traders. As mentioned above, the combination of both features are effective in generating a counter-cyclical uncertainty and those producing larger fluctuations in the stock prices of the firm. This paper provides further foundation for understanding the microfoundations of some of the stylized facts of uncertainty. Indeed, voluntary disclosures respond to positive verifiable manager’s information, as oppose to the disclosures during corporate reporting season (in which disclosures are mandatory, whether these are positive or not). There is a positive correlation between the state of the economy and the voluntary disclosures, which induces the negative correlation with the measure of uncertainty.

This paper builds on the literature of noise rational expectations. In this framework, uncertainty is measured by the conditional variance of the relatively uninformed trader about the liquidation value of the firm. One of the key features of this literature, is that agents make a rational inference about the realization of the liquidation value from the stock price of the firm, while discounting any disclosure appropriately. Moreover, the structure of market for financial information is key. During a recession, the supply of private information is simultaneously less precise, since the manager discloses less information to the markets, and traders rationally reduce the demand for financial information (which increases its price).

The benchmark model presented in the paper is a version of the large-market static noisy rational expectations model standardized by Grossman and Stiglitz [1980], Hellwig [1980], Diamond and Verrecchia [1981], and Admati [1985]. Traders live for one period and exchange the stock of the only firm in the model, which has a random liquidation value, in the asset market. Traders can choose to acquire private information about the liquidation value, which determines their type: informed or uninformed. The optimal price-contingent asset demand schedule is conditioned on the
posterior beliefs of the traders. The presence of noise traders in the asset market (that inelastically demand the shares of the asset), prevent full revelation of the stock price. The traders’ prior of the liquidation value belongs to a bimodal normal distribution, where the two possible values for the mean are perfectly asymmetric, i.e. one the the negative of the other. On the other hand, the manager of the firm receives some private information about the true distribution of the liquidation value (which corresponds to one of the prior value of the trader). In line with Acharya et al. [2011], these event takes place only with some probability, and otherwise the manager shares the same prior to the trader.

While Grossman and Stiglitz [1980] incorporated the incentives for information acquisition into the rational expectation model, endogenously determining the dispersion of beliefs among traders’ types, the relative precision of the each type of trader’s beliefs remain constant over different states of the economy. This result relies on the inelastic supply of information (capture in the constant price of information). Following Veldkamp [2006], we introduce a competitive market for information with free entry, and where information producers face increasing returns of information discovery. In this way, the price for information is also determined endogenously and, in particular, is decreasing with respect to the quantity. The state dependance of the price for information ultimately determines the behavior of the traders’ cross-sectional of beliefs. Finally, the manager of the firm conceals negative information, and pretends to be uninformed.

In terms of the asset pricing, the counter-cyclical behavior of the conditional variance of the uninformed trader generates a positive conditional correlation between the excess return and the expected volatility of the stock. Therefore, the Sharpe ratio for the uninformed (and the representative trader) is also counter-cyclical.

1.1 Empirical Evidence

Most of the literature that analyzes the empirical implications of rational expectations model, used surveys of expectations, such as the Livingston Survey and the Survey of Professional Forecasters, to test the consistency of the professional’s forecasts. This paper is not the exception, and uses the Survey of Professional Forecasters, which records the forecasts of a large number of private
sector forecasters (see Croushore [1993]), to measure the cross-sectional dispersion of beliefs among the professional forecasters.

In particular, while Croushore [2006] finds no bias over the longer sample, he also describes the dynamics of the forecast error as follows: “forecasters go through periods in which they forecast well, then there is a deterioration of the forecasts, and then they respond to their errors and improve their models, leading to lower forecast errors again”. However, he fails to make an assessment about the coincidence of these dynamics with the business cycle.

Figure (1) depicts the dispersion of forecasters beliefs, as described in the appendix (4), over the level of the industrial production (IP) index for the next 6-month horizon. Shaded areas represent recessions, as determined by the National Bureau of Economic Research (NBER). From the figure, it is clear the correlation between the dispersion of forecasts and the NBER recession indicator is significant. In particular, Veronesi [1999] points out that forecasts about the future real output are more disperse when the economy is in a contraction, but fails to provide an foundation for this phenomena. In the same line, Fabio and Antonio [2013] show that financial volatility predicts 30% of post-war economic activity in the United States, and that during the Great Moderation, aggregate stock market volatility explains, alone, up to 55% of real growth.

Most of the empirical literature studying the dynamic behavior of uncertainty, focusses on whether business cycle factors might explain the stock volatility (e.g., Engle and Rangel [2008], Corradi, Distaso, and Mele [2013]). In the same line, Bloom [2009] and Bloom et al. [2012] argue, theoretically and empirically, that uncertainty shocks affect short-run fluctuations in economic activity, and show that uncertainty indexes are negatively correlated with future economic activity, in the short-run. These contributions are complementary to the ones in these paper, as they use uncertainty shocks as the driving process to explain different phenomena, from business cycles to banking panics.

Notably, the Survey of Professional Forecasters allows to classify forecasters by industry, between financial service providers and non-financial service providers. In terms of the framework of the model, the financial service providers are considered to be the informed type of traders, while the non-financial service providers represent the uninformed type of traders.
Figure 1: Dispersion of Forecasts.

Note: Dispersion of forecasts on industrial production. Details on the computation can be found in the Appendix (4). Shaded areas represent recessions as determined by the National Bureau of Economic Research (NBER). Source: Quarterly Survey of Professional Forecasters Philadelphia Fed.
Figure 2: Dispersion of Forecasts.

Dispersion of forecasts on industrial production. Details on the computation can be found in the Appendix. Shaded areas represent recessions as determined by the National Bureau of Economic Research (NBER). Source: Quarterly Survey of Professional Forecasters Philadelphia Fed.
Figure (2) plots the measure for uncertainty for informed and uninformed forecasters. We consider forecasters that belong to financial service providers to be the informed forecasters, while the uninformed are represented by the complement. Details on the computation of the dispersion of forecasts and the classification of forecasters can be found in the Appendix. Conditional on the forecasters type, and in their implied information set, it is clear that the dispersion of informed traders is mostly constant over time, while the dispersion of uninformed traders increases sharply during recessions. Tests on the statistical difference of the dispersion of beliefs among the types of forecasters reject the null hypothesis of equality.

1.2 Literature Review

Economists are still trying to identify the drivers of uncertainty, due to its central role in the financial decision making. Moreover, early in the literature of business cycles (e.g. Pigou [1927], and Clark [1934]) it was emphasized the potentially driving role of uncertainty on the business cycle. Lately, the search for understanding uncertainty has been revitalized, at least partly due to boom and bust cycle of the US economy during the late 2010s, when uncertainty have risen to the levels only comparable to that of the Great Recession.

There is a growing literature that relates to this paper, as it measures uncertainty shocks in various ways. Stock and Watson [2012], Jurado, Ludvigson, and Ng [2013] and Bachmann et al. [2013] document the properties of uncertainty for the U.S., some emerging economies and Germany (for the latter paper).

While the asset price volatility is empirically correlated with the state of economic conditions, the financial literature has not yet provided an analytical model to capture such link. Schwert [1989] early pointed out the link between stock market volatility and the business cycle. However, as noted by David and Veronesi [2013], the relationship is quiet complex and goes beyond the simple boom-bust cycle variation. Linear regressions of stock market volatility on virtually all the macroeconomic variables that have been tested in the literature, accounts for only a small proportion of stock market volatility.

A recent view in macroeconomics and finance stresses the importance of the beliefs heterogene-
ity, and the information dispersion, as one of the common feature of financial markets. In Albagli, Hellwig, and Tsyvinski [2011], heterogeneous information provides a natural source of excess price volatility, since the marginal trader pricing the asset overweights his own private information relative to the market. Biais, Bossaerts, and Spatt [2010] show how different information sets produce different portfolios, and portfolio separation does not obtain, which enables investors to outperform the index. However, the literature on asymmetric information has mostly concentrated on models that examine the implication of passive learning, and avoided to reproduce the dynamics of information dispersion and disagreements, among the agents in the market, over the business cycle.

The contribution of this paper is to develop a theoretical model that captures the cyclical behavior of asset prices and uncertainty during the business cycle, that capture the role of informational frictions. The paper provides further evidence on the dynamic behavior of beliefs, measured by the survey of professional forecasters. The dispersion of forecasts increases during recessions and retracts during booms. The model shows the importance of the manager’s disclosure decision, that increases the precision of the the information privately distributed, and thus the incentives for investors to acquire information.

The objective of this article is to investigate the dynamics of asset pricing, investors’ conditional variances and expectations in a noisy dynamic rational expectations asset pricing equilibrium model. The model features several components: (i) the manager randomly receives private information and decides whether to disclose it, (ii) information producers decide whether to discover information facing a fixed cost of information discovery, and (iii) investors decide whether to acquire private information and their assets demands. Some investors have private information about the future cash flows of the asset, while others remain relatively uninformed. Prices are only partially revealing once we allow for the presence of noise traders in the asset market.

The equilibrium prices can be represented by a representative investor that is condensed by a weighted average of the moment conditions of the informed and uninformed investors. The moment conditions are weighted using the proportion of types of investors in the market, which fluctuates during the business cycle. In recession, the conditional moments of the uninformed trader are more
heavily represented, while the opposite is true during expansions.

The rest of the paper is organized as follows. In section (2) I present the setup of static the model. Finally, in section (3) I present some final remarks.

2 A Static Model of the Information Market

This section presents a static model of the information market. Building on Acharya et al. [2011], the model considers the disclosure decision of the manager of the single firm in the economy. His disclosure improves the precision of the signal supplied by information producers in the information market, which are modeled following Veldkamp [2006]. Finally, in line with Grossman and Stiglitz [1980], ex-ante identical traders can choose to acquire signals in the information market before submitting their demands for the shares of the single firm.

2.1 Setup

The model has three stages, indexed by $t \in \{0, 1, 2\}$, and the economy is populated by a manager, traders and information producers. There are two assets: a riskless asset with payoff normalized to 1, and a risky asset representing the firm’s shares. The risky asset is in zero net supply, and $\theta$ represents its liquidation value paid to the shareholders at $t = 2$. All agents share the same prior about $\theta$, which is assumed to be distributed according to a normal distribution:

$$\theta \sim \mathcal{N}(M, \tau_\theta^{-1}),$$ \hspace{1cm} (1)

where the unconditional mean can take two possible values, $M \in \{-\mu, \mu\}$, with equal probability. The unconditional distribution of $\theta$ is given by $f(\theta)$, defined below.

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2This assumption departs from Grossman and Stiglitz [1980] who proposed that the asset payoffs have a learnable component and an idiosyncratic component; the learnable component is perfectly observed at the private signal is revealed. This model considers the case of noisy private signal and non-perfectly learnable dividend payoffs.
\[ f(\theta) = \frac{1}{2} \phi(\tau(\theta - \mu)) + \frac{1}{2} \phi(\tau(\theta + \mu)) \]  

(2)

where \( \phi(\cdot) \) represents the PDF of the standard normal distribution. Figure 3 shows the prior distribution of \( \theta \) shared by all the agents.

The manager runs the single firm in the economy, and may learn some relevant information about the liquidation value of the firm. The manager can choose to disclose this information to the information producers, or can conceal it. The shares of the firm are traded in the asset market by two types of traders: a continuum of risk-averse traders, indexed by \( i \in [0, 1] \), and a stochastic measure of liquidity traders denoted by \( \Phi(u) \), where \( \Phi(\cdot) \) represents the CDF of the standard normal distribution. Before

Risk-neutral traders can acquire relevant information about the liquidation value of the firm from the information producers, in the information market. There is a continuum of information producers, and the information market is a competitive market.

Traders share the same prior about \( \theta \), which is distributed according to (3), where \( f(\theta) \) repre-
sents the pdf of $\theta$, and $\phi(\cdot)$ the pdf of the standard normal distribution. However, the manager might receive privileged information about the realization of $\theta$, and learn whether the realization of $\theta$ was drawn from a normal distribution with mean $\mu$ or $-\mu$.

$$f(\theta) = \frac{1}{2}\phi(\tau_\theta(\theta - \mu)) + \frac{1}{2}\phi(\tau_\theta(\theta + \mu))$$ (3)

Each of the normal distributions combined in (3) has a mean given by $\mu$ and $-\mu$, respectively, and unconditional variance $\tau_\theta^{-1}$. Moreover, the traders’ prior probability of drawing $\theta$ from each distribution is given by $\frac{1}{2}$.

2.1.1 Firm’s Manager

The objective of the manager is to maximize the market value of the firm (at the trading stage, at $t = 1$), capture by the price of the risky asset $p$. In the spirit of Acharya et al. [2011], at the beginning of the game with probability $\kappa$ the manager perfectly learns the realization of $\mathcal{M}$, from equation (1), i.e. whether $\theta$ was drawn from the normal distribution with unconditional mean $\mu$ or $-\mu$.

The event where the manager is informed remains private only with probability $\nu$. In that case, the manager can voluntary disclose his information or conceal it. Otherwise, with probability $1 - \nu$ the manager is publicy informed, and does not have discretion over the information, i.e. the manager immediately reports his private information. The manager has discretion only with probability $\kappa\nu$. In either case, the manager discloses information only to the information producers, who act as intermediaries between the manager and the traders.

The disclosure is verifiable by the information producers and cannot be manipulated. The manager’s action, to disclose either voluntarily or compulsorily, becomes common knowledge to the agents. However, traders cannot distinguish what was reason of the disclosure. The information disclosure increases the precision of the signal supplied by the information producers.

Denote $\delta \in \{0, 1\}$ the manager’s decision whether to disclose the information or not, where $\delta = 1$ if the manager discloses (irrespective if is voluntarily or compulsorily).

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3Assuming “verifiable reports” is common to the literature. See, for example, Shin [2003].


2.1.2 Information Producers

After the manager’s decision, each information producer can choose to perfectly discover the realization of $\theta$ by paying a fixed cost $\chi^4$. The information producer’s profits depend on the price charged for each private signal supplied, as well as the pricing strategies of other information suppliers - which determined the residual information demand for each information producer$^5$.

After discovering $\theta$, each information producer supplies unbiased signals of $\theta$ in the market for financial information. Each signal contains an idiosyncratic noise which is independent from one another, but ex-ante identical. This assumption can be motivated by the idiosyncratic news interpretation device each trader uses. Moreover, there is no replication cost of the information, i.e. the marginal cost of each signal is zero.

Following Veldkamp [2006], the perfectly competitive market for information has two important features. There is free entry in the market for information, so the incumbent producers are forced to have zero profits, and traders are prevented from reselling any purchased information.

Denote $\tau_{\epsilon}(\delta)$ the precision of the idiosyncratic private signal, conditional on $\theta$, which is assumed to depend on the action of the firm’s manager. This assumption can be motivated by the fact that when the manager releases information to the information producer, the quality of the signal produced increases while it becomes easier for the trader to analyze and process the content of the private signal. The value of $\tau_{\epsilon}(\delta)$ increases when the manager disclose his private information to the information suppliers,

$$
\tau_{\epsilon}(\delta) = \begin{cases} 
\tau_s + \tau_f & \text{if } \delta = 1 \\
\tau_s & \text{if } \delta = 0 
\end{cases}
$$

(4)

Denote $\lambda(\delta)$ the aggregate demand for information, and $\lambda_j(\delta)$ the demand for information received by the information producer $j$. Moreover, let $\psi_j = 1$ denote the information producer

$^4$The fixed cost of discovery assumption follows Veldkamp [2006] and generates increasing returns to the information producer which in turn generates a decreasing supply schedule - as oppose to the constant price generate by a constant return assumption.

$^5$One way to ensure that the market is contestable is to force information suppliers to choose prices in a first stage and choose entry in a second stage. This is a reasonable way to think of news markets where the price of the periodical is fixed well in advance and then editors decide whether or not to supply a story.
j’s decision to discover information, and \( \psi_j = 0 \) otherwise, and \( c(\lambda) \) the market price for financial information for a given decision of the manager. The entry problem faced by the information producer \( j \) is

\[
\max_{\psi_j \in \{0,1\}} \psi_j (c(\lambda) \lambda_j (\delta) - \chi)
\] (5)

The optimal entry policy of the information producer is described by a corner solution, \( \psi_j(\lambda) : [0,1] \rightarrow \{0,1\} \).

2.1.3 Risk-Averse Trader

Each risk-averse trader initially receives a random endowment of shares of the risky asset, represented by \( e \), which could be thought as including labor and other sources of income. The endowment is distributed according to a normal distribution with mean zero and variance \( \tau_e^{-1} \), and independent across investors: \( e \sim \mathcal{N}(0, \tau_e^{-1}) \).

The risk-averse trader has constant absolute risk aversion (CARA) preferences \(^6\) over the final wealth, with (positive) risk aversion parameter \( \rho \). The expected utility of the risk-neutral trader is

\[
U(w) =\mathbb{E}[-\exp(-\rho w) \mid \mathcal{I}]
\] (6)

where \( w \) denotes his final wealth, and \( \mathcal{I} \) denotes his information set after the information acquisition decision.

Each trader makes a rational inference about \( \theta \) from the realization of the market price. Furthermore, in the spirit of Grossman and Stiglitz [1980], the trader decides whether to acquire further information about \( \theta \) in the market for financial information, i.e. purchasing an idiosyncratic private signal at the competitive price; prior to making the portfolio decision. The information acquisition decision, the trader weights the benefit between obtaining the private signal at a cost \( c(\lambda) \), or simply extracting information from the asset price. Let \( \iota(\lambda, \delta, p) : [0,1] \times \{0,1\} \times \mathbb{R} \rightarrow \{0,1\} \) denote the trader’s decision, where \( \iota = 1 \) represent the action to purchase the private signal.

\(^6\)Note that given that investors have CARA preferences, the optimal portfolio decisions are independent of the initial wealth, therefore of the endowments.
The private idiosyncratic signal purchased by the informed trader is denoted by $s$, and distributed according to a normal distribution with mean $\theta$ and unconditional variance $\tau^{-1}_\theta + \tau^{-1}_\epsilon$,

$$s = \theta + \epsilon$$

where the idiosyncratic noise $\epsilon$ is distributed according to a normal distribution with mean zero and precision $\tau_\epsilon$. Recall that equation (4), above, describes the two possible values for $\tau_\epsilon$; the higher value being achieved upon the manager’s disclosure.

Conditioning on the information acquisition decision, there are two types of risk-averse traders: informed traders whose information set is given by the private signal and the asset price $\{s, p\}$, and uninformed traders that only learn about $\theta$ from the asset price, and then have an information set given by $\{p\}$. The proportion of informed traders, denoted by $\lambda = \sum_j \lambda_j$, is endogenously determined by the market clearing condition in the market for financial information. The information set of the trader, after the information acquisition stage - upon the determination of $\iota \in \{0, 1\}$, is

$$\mathcal{I} = \begin{cases} 
\{p\} & \text{if } \iota = 0 \\
\{p, s\} & \text{if } \iota = 1
\end{cases}$$

Note that there are two sources of heterogeneity for risk-averse traders: the endowment of risky asset $e$, and the realization of the private signal.

The final wealth of the trader is given by

$$w = x(\theta - p) + ep - ic$$

where $x$ denotes the trader’s position on the risky asset, and $c$ denotes the competitive market price of the private signal. As mentioned before, the return on the riskless asset is normalized to zero.
2.1.4 Noisy traders

The stochastic measure of noisy traders demand the risky asset inelastically with respect to the price $p$. Following Grossman and Stiglitz [1980], the role of the noise traders is to prevent the asset price from perfectly revealing the realization of the fundamentals, condensed in the aggregate demand from informed investors. In line with Albagli et al. [2011], the measure of noise traders is given by $u$ which is normally distributed, independently of $\theta$, according to

$$u \sim N(0, \sigma_u^2)$$

(8)

2.1.5 Timing

The firm’s manager, the information producers and the risk-averse traders play the following game. The order of events is summarize in the Figure (5). At the beginning of period $t = 0$, the manager decides whether to disclose the private information about the true distribution of $\theta$, to the information producers, increasing the precision of the signal supplied by the latter ones. Then, at the end of period $t = 0$, information producers decide whether to participate in the market for information by paying the fixed cost $\chi$ to discover $\theta$. At the beginning of period $t = 1$, each risk-averse trader decides whether to acquire information or not, at the market price for information, and at the end of period $t = 1$, each trader solves the optimal portfolio problem conditioning on his information set. Finally, at $t = 2$ the asset pays off.

All aspects of the model except for $\{\theta, \{\epsilon_i\}_i, u\}$, and the event that the manager is informed, are common knowledge. Random variables $\{\theta, \{\epsilon_i\}_i, u\}$ are uncorrelated with each other. Sufficient assumptions are make such that, the average signal coindices with the true realization of $\theta$, $\int_0^\lambda s_i di = \theta$ almost surely (a.s.) (i.e., $\int_0^\lambda e_i di = 0$). The actions of every agent are private, expect for the manager’s disclosure.
Figure 4: Firms’ manager idiosyncratic states.

Note: With probability $\kappa$, the manager exogenously observes $s^F$. If the manager is informed, with probability $\nu$ the event remains private information for the manager. In this case, the manager has discretion regarding voluntary disclosing the signal. With probability $\kappa(1 - \nu)$ the manager is “publicly” informed and so does not have discretion over the information’s release. In this case, the manager reports it immediately to the market.

Figure 5: Timing of the game
2.2 Market for Information

Abstracting from the manager’s decision - to disclose or to conceal his private information - there are two opposite tensions influencing the trader’s decision to acquire information. On one hand, private information is a strategic substitute, in the sense that it is more valuable the less informed traders there are in the market. This is the effect described by Grossman and Stiglitz [1980], in terms of this model: the closer $\lambda$ is to 1, the information about $\theta$ revealed through the asset price is relatively higher, and the incentives to acquire information are lower.

On the other hand, the increasing returns to scale in the information discovery process augment the strategic complementarity of information, since signals become cheaper the higher is $\lambda$. Veldkamp [2006] was the first to introduce this feature into the market for financial information. These opposing forces can potentially generate multiple equilibria in the market for information, but there is only one stable equilibrium with positive information production. However, these effects are not enough to generate changes in the conditional variance of traders across different aggregate states.

In order to show the limited scope of these effects, figure (6) depicts the equilibrium in the market for information holding constant the precision of the private signal. The vertical axis measures the information demand (which depends on the relative precision of the two types of traders, i.e. the precision of the informed relative to the uninformed trader denoted by $\tau_I/\tau_U$), and the information supply which depends on the average cost of information. As long as the precision of the private signal $\tau_\epsilon$ is independent of the $\theta$, so will be the relative precision of the informed and uninformed traders.

The left panel of figure (6) displays the information market assuming a constant price for information, à la Grossman and Stiglitz [1980], while the right panel displays the market assuming increasing return in the information discovery and therefore a decreasing supply schedule of information, à la Veldkamp [2006]. The solid lines displays the supply schedule, and the dashed lines represent the demand for information, which is decreasing function of the the marginal benefit of acquiring information captured by $\tau_I/\tau_U$.

The equilibrium in the market for information is determined by the intersection of the demand and the supply. The left panel is characterized by a unique stable equilibrium with strictly pos-
itive information production. The equilibrium is denoted by $A$, and determines the proportion of informed investors $\lambda$. The right panel features multiple equilibria, but only one of those is a stable equilibrium with strictly positive information supply: point $A$. Point $B$ denotes the unstable equilibrium: a marginal increase in $\lambda$ reduces the marginal cost more than the marginal benefit and therefore an uninformed investor can benefit from purchasing the signal. Finally, point $C$ is an stable equilibrium with no information supply.

To generate changes in the relative conditional variance of traders, either the precision of the dividends $\tau_0^7$, the precision of the private signal $\tau_e$ or the variance of the measure of noise traders $\tau_u^{-1}$ must fluctuate with the aggregate state. In this paper, the change in the relative conditional precisions is caused by the manager’s actions: the disclosure of the manager increases $\tau_e$ as described by equation (4).

Figure (7) extends the previous results by considering the manager’s decision. There are two dashed lines for each panel, representing different information demands. The lower dashed line coincides with that of figure (6), when $\tau_e$ takes its lower value. The upper dashed line represents the marginal benefit of acquiring information when the manager discloses his signal and $\tau_e = \tau_s + \tau_f$.

Again, the left panel assumes a constant cost of information while the right panel displays the information market assuming increasing returns in the information production. In either case, the marginal benefit of acquiring information increases upon a higher precision signal. Therefore, each stable equilibrium with strictly positive information supply has a higher proportion of informed investors. However, the relative conditional precision of traders in equilibrium for the case of a constant cost of information remains unchanged, which is not supported by the empirical evidence shown in section (1). For this reason, we consider in this model the case where the information price is endogenous and information producers face increasing returns.

### 2.3 Asset Market

Assets are traded in a market with publicly observable market-clearing prices. The riskless asset price is normalized to 1 and the market clearing price for the risky asset is denoted by $p$. Using

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7Veldkamp [2006] assumes a countercyclical behavior for $\tau_0$, generated by a Geometric Brownian process of dividends.
Figure 6: Market for financial information with constant marginal cost.

The figure displays the information market closing down the manager’s decision: \( \tau_c = \tau_s \). Panel (a) displays the information market assuming constant fixed cost of information, à la Grossman and Stiglitz [1980]: the equilibrium is unique. Panel (b) assumes increasing returns in information discovery, à la Veldkamp [2006]: there are multiple equilibria, but only point \( A \) is an stable equilibrium with positive information supply.

Figure 7: Market for financial information with decreasing marginal cost.

The figure displays the information market including the manager’s decision: \( \tau_c \in \{ \tau_s, \tau_s + \tau_s F \} \). Panel (a) displays the information market assuming a constant fixed cost of information, à la Grossman and Stiglitz [1980]: the equilibrium is unique. Panel (b) assumes increasing returns in information discovery, à la Veldkamp [2006]. The solid line displays the marginal cost of information, the lower dash line represents the marginal benefit of information without disclosure of the manager and the upper dash line represents the marginal benefit of information with disclosure of the manager. The underlying assumption is that disclosure occurs when aggregate state is higher.
well established results for large markets, the optimal strategy of each trader is to submit an asset demand schedules contingent on his information set and the current price, along with the result from the information market and the manager’s disclosure (but for brevity, they are omitted from the description of control variables). Let $x(s,p) : \mathbb{R}^2 \to \mathbb{R}$ denote the demand schedule for the risky asset of an informed trader whose observes a private signal $s$, and $x(p) : \mathbb{R} \to \mathbb{R}$ denotes the demand schedule of an uninformed trader. For simplicity, the set of informed traders is sorted in the interval $[0, \lambda]$, while the measure $1 - \lambda$ of uninformed traders are sorted in the interval $(\lambda, 1]$.

Equation (9) below describes the limit order book for the risky asset. It considers the position of the informed and uninformed traders, as well as the demand from the noise traders $u$ and the supply of the risky asset coming from the idiosyncratic random endowments $e = \int_0^1 e_i di$, assumed equal to 0. From equation (9), it is clear the asset price function $p : \mathbb{R}^3 \to \mathbb{R}$ depends explicitly on $\lambda$, $u$ and the composite of idiosyncratic signals $\int_0^\lambda s_i di = \theta$, and implicitly on $\delta$.

$$L(p) = \int_0^\lambda x_i(s_i,p)di + \int_\lambda^1 x_i(p)di + u - \int_0^1 e_i di = 0 \tag{9}$$

As is the case in most of the literature, attention is restricted to linear equilibria, i.e. linear asset price functions in the fundamental $\theta$ and $u$. Equation (10) describes the guess for the asset price, where the coefficients depend on the endogenous variable $\lambda$. In particular, $A(\lambda) = 0$ is decreasing in $\lambda$, and $B(\lambda)$ and $C(\lambda)$ are monotonically increasing in $\lambda$.

$$p = A(\lambda) + B(\lambda)\theta + C(\lambda)u \tag{10}$$

Conditioning on $\lambda$, this guess implies that the asset price is distributed according to a normal distribution, which in turn delivers a linear schedules for the trader’s assets demands.

The equilibrium in the asset market is solved using the guess and verify method. While uninformed investors do not observe any private signal, they are able to extract an unbiased noisy signals of $\theta$ from $p$. The next lemma characterize the public signal embedded in $p$.

**Lemma 1.** Let $z = \theta + \alpha(\lambda)u$ be the public signal embedded in the asset price, where $\alpha(\lambda) = C(\lambda)/B(\lambda)$. Then, the public signal is an unbiased signal of $\theta$, and is distributed according to
a normal distribution with mean $\theta$ and conditional variance $\alpha(\lambda)^2\tau_u^{-1}$. Moreover, $p$ and $z$ are observationally equivalent.

Proof. Define the market-clearing price function $p(z) = B(\lambda)z + A(\lambda)$. The price function is invertible, and satisfies the asset pricing rule described by (10). The fact that traders know $\lambda$ prior to engaging in the asset trading, allow them to use the pricing rule to extract only the relevant information, concerning $\theta$. Conditioning on $\{p, \lambda\}$, by observing the limit order book (9), the trader learns $z$. In particular, it will be shown later that $\alpha(\lambda)$ is monotonically increasing in $\lambda$. □

Lemma (1), above, shows that, given the assumption with respect to the price linearity, the public signal $z$ contains all the relevant information embedded in the market asset price. Therefore, the use of $p$ or $z$ in the conditioning set of traders to capture the public information contained in the market asset price, is interchangeable. In particular, conditional on $\theta$, the public signal has a precision $\tau_z = \alpha(\lambda)^2\tau_u$.

2.4 Equilibrium

This section defines the equilibrium concept for the benchmark setting. Before proceeding, the remaining elements of the equilibrium are defined. The risky asset price function is given by

$$p(\lambda, \delta, \theta, u) : [0, 1] \times \{0, 1\} \times \mathbb{R}^2 \rightarrow \mathbb{R}.$$ 

Definition 1. Given the random endowments, the fundamental payoff, and the noisy traders’ asset demand: $\{\{e_i\}, \theta, u\}$, a Perfect Bayesian Equilibrium consists of an asset demands functions for informed traders $x(s, p)$, and for the uninformed trader $x(p)$, a price function $p(\theta, \lambda)$, information demands $i(\lambda, \delta, p)$, information supply $\psi(\delta)$, information price $c(\lambda)$, managers’ disclosure decision $\delta$ and posterior beliefs for the investors about the managers’ information status $H(\cdot)$ such that (i) $x(s, p)$ is optimal given $H(\theta | s, p)$ and $x(p)$ is optimal given $H(\theta | p)$; (ii) the asset market clears for all $(\theta; u)$; (iii) a fraction $\lambda$ of traders purchase information given $H(\cdot)$; and (iv) $H(\cdot)$ satisfies Bayes’ rule whenever applicable.
2.5 Solving the Model

The equilibrium is computed by backward induction starting at the end of period \( t = 1 \), describing the traders’ optimal solution for the portfolio choice decision first, and information acquisition problems later. Then, the optimal information supply of information producers at the end of period \( t = 0 \) is computed, and finally the manager’s optimal disclosure decision is described at the beginning of period \( t = 0 \).

2.5.1 Risk-averse Investor

Let \( V(p) \) be the expected utility of a trader, prior to observing any private signal and after observing an asset price \( p \). Equation (11) describes the trader’s problem.

\[
\begin{align*}
V(p) = \max \left\{ \mathbb{E} \left[ V^I(s, p) \mid p \right], V^U(p) \right\} \\
s.t. \\
V^I(s, p) = \max_x \mathbb{E} \left[ -\exp(-\rho w) \mid s, p \right] \\
V^U(p) = \max_x \mathbb{E} \left[ -\exp(-\rho w) \mid p \right] \\
w = x(\theta - p) + cp - \iota c
\end{align*}
\]

(11)

\( V^I(s, p) \) represents the expected utility for a trader who acquired a private signal \( s \), and follows the optimal investment strategy. The expectation is conditioned only on the asset price: prior to the realization of the signal. Analogously, \( V^U(p) \) refers to the expected utility of the uninformed trader, who follows the optimal investment strategy. The outer maximization is with respect to the information choice, captured by \( \iota \). Optimality requires the trader to be indifferent between both choices.

**Portfolio Choice.** Given that the trader has a non-normal prior over \( \theta \), the optimal portfolio problem is solved using an alternative approach relative to the most of the literature on noisy rational expectations. In particular, this solution relies in the following result that proves that maximizing a CARA utility function is equivalent to minimizing the cumulant generating function (CGF) with respect to the distribution of the excess return \( \theta - p \).

**Proposition 1.** The objective function of an agent with CARA utility that solves (6) with respect
to $x$ and subject to (7), is equivalent to solve

$$\min_x \text{CGF}_{\theta-p}(-\rho x)$$

where $\text{CGF}_{\theta-p}$ represents the cumulant generating function of $\theta - p$, conditional on $p$. In particular, the CGF using the pdf from equation (3), is given by

$$\text{CGF}_{\theta-p} = \log \left[ e^{\frac{x\mu(2\mu^2\rho - 2\rho \mu + x \rho)}{2\tau_\theta}} \left( 1 - \pi + \pi e^{2x \mu \rho} \right) \right],$$

where $\pi(\delta) = \text{Prob}[M = \mu | \delta]$ captures the trader’s posterior probability that the unconditional mean of $\theta$ is given by $\mu$, upon observing the manager’s disclosure - which is reflected in precision of the private signal. Note that $\pi(\delta)$ coincides for both types of traders. The CGF coincides with that of a zero-mean normal distribution as $\mu \to 0$.

**Proof.** See Davila [2011], proposition 1.

In line with the case of normal distribution, the CARA trader facing a bimodal normal distribution considers only first two moments of the distribution: the mean and variance, which depend on $\pi$ and according to the prior are given by $(1 - 2\pi)\mu$ and $\tau_\theta^{-1} + \pi(1 - \pi)\mu^2$, respectively.

The symmetric information structure across each type of trader generates demand schedules that are also symmetric across types. Denote $\tau_I = \tau_\theta + \tau_z + \tau_\epsilon$ and the precision of the informed trader by $\eta_I = \text{var}(\theta \mid s, p)^{-1}$. Analogously, the precision of the uninformed trader is denoted by $\eta_U = \text{var}(\theta \mid p)^{-1}$, and $\tau_U = \tau_\theta + \tau_z$. Given the posterior of the traders, the conditional precision, for each type of trader, is given by

$$\eta_h = \frac{\tau_h^2}{\tau_h - 4\tau_\theta^2 \mu^2 (\pi - 1)\pi}, \quad h \in I, U$$

Note that the conditional precisions depends on $\mu$ and $\pi$, since it determines the dispersion between the two underlying distribution embedded in $\theta$.

The conditional distribution of $\theta$ for the uninformed trader whose information set only contains the market price, denoted by $f^U(\theta \mid z)$, is
The risky asset demand schedule for the uninformed trader is given by equation (12), where
the last expression is approximated using a first order Taylor approximation around $x_0 = 0^8$. Note
that given the perfectly asymmetric dependance of the normal distributions on the unconditional
mean $\mu$, the optimal portfolio coincides with the canonical solution for the CARA-normal setup
when $\mu \to 0$.

$$x(p) = \arg\min_x CGF_{f(\theta - p | z)}(-\rho x) \approx \frac{z\tau_z - p\tau_U - \tau_\theta \mu (1 - 2\pi)}{\rho}$$ (12)

It is worth emphasizing that $x(p) = \mathbb{E}[\theta - p | z] \times \tau_I$.

The conditional distribution about $\theta$ of the informed trader, whose information set contains
both $z$ and the private signal $s$, denoted by $f^I(\theta | s, z)$, is equal to

$$f^I(\theta | s, z) = \pi(\delta)\phi\left(\tau^I \left(\theta - p - \frac{\tau_\theta \mu + \tau_z z + \tau_s s}{\tau_I}\right)\right) + (1 - \pi(\delta))\phi\left(\tau^I \left(\theta - p - \frac{-\tau_\theta \mu + \tau_z z + \tau_s s}{\tau_I}\right)\right),$$

where, the informed trader puts less weight on the unconditional mean $\mu$ relative to the unin-
formed trader.

Following analogous steps as for the uninformed trader, the optimal risky asset demand schedule
for the informed trader is describe in equation (13).

$$x(s, p) = \frac{z\tau_z + s\tau_\epsilon - p\tau_I - \tau_\theta \mu (1 - 2\pi)}{\rho}$$ (13)

In line with the linear asset pricing rule from equation (10), demands are also linear on prices
and signals. Consider the following representations of the traders’ demands: the demand from

---

8This assumption is without loss of generality, but allows for a analytical solution. Approximating the expression
with a higher order Taylor approximation, or even around a non-zero value for $x$, does not change the quantitative
results.
the informed that observes signal $s$, is an given by $x(s,p) = as + \varsigma^I(p,\lambda)$, where $a$ measures the trading intensity of informed investor, and $\varsigma^J(p,\lambda)$ for $j = \{I, U\}$, is a linear function on prices but potentially non-linear on $\lambda$. The guess for the demand of the uninformed agent is $x(p) = \varsigma^U(p,\lambda)$. Then, replacing the latter expressions into the noisy limit order book from equation (9), and using the fact that $\int_0^\lambda s_i di = \theta$ a.s., the market clearing condition reduces to

$$L(p) = \int_0^\lambda (as_i + \varsigma^I(p,\lambda)) di + \int_\lambda^1 \varsigma^U(p,\lambda) di + u = \lambda a \varsigma + \varsigma(p,\lambda) = 0$$

where $\varsigma(p,\lambda) = \lambda \varsigma^I(p,\lambda) + (1-\lambda) \varsigma^U(p,\lambda)$ is a linear function of the price. The public signal revealed from the limit order book (in particular, the intercept of the limit order book) is given by

$$z = \theta + \frac{u}{\lambda a}$$

which, in line with the lemma (1), $\alpha(\lambda) = \lambda a^{-1}$.

The next proposition (2) shows that existence of a unique equilibrium in the asset market. In particular, the equilibrium is such that as $\mu \to 0$ it converges to the analogous equilibrium using a normal distribution prior (isolating from the manager’s disclosure decision).

**Proposition 2.** *(Asset market equilibrium)* Given the fraction of informed traders $\lambda$ *(the equilibrium in the market for financial information)* and the manager’s disclosure decision $\delta$ *(which determines the precision of the private signals)*, there exists a linear asset price $p(z,\lambda,\delta)$ that clears the asset market *(solves equation (9).* ) The equilibrium in the asset market is described by the investors’ asset demands,

$$x(s,p) = as + \varsigma^I(p,\lambda), \quad \text{with} \quad a = \frac{\tau_z}{\rho}, \quad \varsigma^I(p,\lambda) = \frac{(z\tau_z - p\tau_I - \tau_0\mu(1-2\pi))}{\rho}$$

and market clearing price given by,

$$p(z,\lambda,\delta) = \frac{z(a\lambda \rho + \tau_z) - \tau_0 \mu(1-2\pi)}{\lambda \tau_I + (1-\lambda)\tau_U}$$
Proof. Solving for the market clearing condition (9) implies:

\[ 0 = a\lambda z + \lambda^{-1} (z\tau_z - p\tau_I - \tau_\theta \mu (1 - 2\pi)) + (1 - \lambda)\rho^{-1} (z\tau_z - p\tau_U - \tau_\theta \mu (1 - 2\pi)) \]
\[ = \rho a\lambda z + z\tau_z - \tau_\theta \mu (1 - 2\pi) - p(\lambda\tau_I + (1 - \lambda)\tau_U) \]

This yields the market-clearing price as given by equation (15). In terms of the guess (10),

\[ A(\lambda) = \frac{-\tau_\theta \mu (1 - 2\pi)}{\lambda\tau_I + (1 - \lambda)\tau_U} \]
\[ B(\lambda) = \frac{a\lambda\rho + \tau_z}{\lambda\tau_I + (1 - \lambda)\tau_U} \]
\[ C(\lambda) = a\lambda \frac{a\lambda\rho + \tau_z}{\lambda\tau_I + (1 - \lambda)\tau_U} \]

\(\square\)

Consistently with the initial guess about the asset price, the equilibrium price function is linear in the public signal. Below, figure (8), displays the price function for different realizations of the precision of the private signals \(\tau_\epsilon \in \{\tau_s, \tau_s + \tau_f\}\).

Define the average expectation of the informed traders as \(E^I(\theta | \tilde{s}, p) \equiv \lambda^{-1} \int_0^\lambda E(\theta | s, p) \, d\theta\). Then, the average expectation of traders is \(E^A(\theta | I^A) \equiv \omega E^I(\theta | \tilde{s}, p) + (1 - \omega)E(\theta | p)\) where \(\omega \equiv (\lambda\tau_I + (1 - \lambda)\tau_U)^{-1}(\lambda\tau_I)\). The average precision is \(\tau^A \equiv (\lambda\tau_I + (1 - \lambda)\tau_U)\). The equilibrium price is the \(\omega\)-weighted expectation of the liquidation value of the firm plus a noise term proportional to \(u\),

\[ p = \omega E^I(\theta | \tilde{s}, p) + (1 - \omega)E(\theta | p) + \frac{\gamma u}{\lambda\tau_I + (1 - \lambda)\tau_U} \]
\[ = E^A(\theta | I^A) + \gamma u(\tau^A)^{-1} \]

The equilibrium prices is identical to those that would obtain in a homogeneous information-representative agent economy, where the representative trader would have beliefs according to \(E^A\) and \(\tau^A\).
2.5.2 Information Acquisition

This subsection studies the information acquisition problem of the trader, at the beginning of \( t = 1 \). Each trader chooses his own type, by determining whether to become informed or remain relatively uninformed. From a game theoretic perspective, the presence of noise traders prevents the asset price from fully revealing the private signals. It subsequently prevents uninformed traders from pretending to mimic the other type of trader.

Each trader weights paying the price of information \( c \), which is taken as given, and the relative benefit of observing the private signal. If \( \lambda \in (0, 1) \) in equilibrium, there exists a marginal trader that is indifferent between both types. In terms of the model, in equilibrium the marginal traders satisfies \( V_U(p) = \mathbb{E}[V^I(s, p) \mid p] \). These equilibrium can be interpreted as if the marginal trader plays a mixed strategy, and becomes informed with probability \( \lambda \).

The expected utility of the uninformed trader \( V^U(p) \) is obtained by replacing the optimal investment decision from equation (12) into the expected utility \( \mathbb{E}(\exp(-\rho(x(p)(\theta - p) + e_ip)) \mid z) \), and is equal to

\[
V^U(p) = -\mathbb{E}[-\exp(-(z\tau_z - p\tau_U - \tau_\theta\mu (1 - 2\pi))(\theta - p)) \mid z] \exp(-\rho e p), \tag{19}
\]

where the last term represents the utility derived from the endowment.

The analytical solution for \( V^U(p) \) corresponds to a weighted average (with weights equal to \( \pi \) and \( 1 - \pi \), respectively) of the Moment Generating Function (MGF) at \( x(p) \), of the normal distributions according to a mean \( \mu \) and \( -\mu \) (and conditional variance given by \( \eta^U \)), respectively.

On the other hand, the expected utility for the informed trader, given the information set \( \{s, p\} \), is computed by replacing (13) into \( E(\exp(-\rho(x(s,p)(\theta - p) + ep)) \mid s, z) \), and is equal to

\[
V^I(s, p) = -\mathbb{E}[\exp(-(z\tau_z + s\tau_e - p\tau_I - \tau_\theta\mu (1 - 2\pi))(\theta - p)) \mid s, z]\exp(-\rho(ep - c)),
\]

where the last term contains the utility derived from the endowment minus the cost of acquiring the private signal.
Note that the informed trader takes the decision whether to acquire information prior to observing the private signal, \(s\). Therefore, the relevant moment generating function for determining the indifference condition for the marginal trader, is taking with respect to the same information set as the uninformed type,

\[
\mathbb{E}[V^I(s) \mid z] = \mathbb{E}[\mathbb{E}[\exp(-(z\tau_z + s\tau_\epsilon - p\tau_I - \tau_\theta\mu(1 - 2\pi)(\theta - p)) \mid s, z] \mid z] \exp(-\rho(ep - c))
\]

The ex-ante expected utility of the informed trader, as shown in equation (20) adjusts that of the uninformed trader by: (i) introducing a term that measures the relative precision for each type of traders (that accounts for the marginal benefit) and, (ii) includes the marginal cost of information.

\[
\mathbb{E}[V^I(s,p) \mid z] = \sqrt{\frac{\tau_U}{\tau_I}} \frac{V_U(p)}{V_I(p)} \exp(-\rho(ep - c)) (20)
\]

For the marginal trader to be indifferent for either type, the ratio of expected utilities between the different types (19) and (20) must equal 1,

\[
1 = \frac{\mathbb{E}[V^I(s,p) \mid z]}{V^U(p)} = \sqrt{\frac{\tau_U}{\tau_I}} \exp(\rho c) (21)
\]

Equation (21) yields the rule for the optimal decision of the trader in terms of the individual demand for information. In particular, the trader demands information as long as the cost for the private signal falls below a monotone function of the ratio of posterior precisions of the informed and uninformed types. In particular, the individual demand for information is given by

\[
i(\lambda, \delta) = \begin{cases} 
1 & \text{if } c \leq \frac{1}{2\gamma} \ln \left( \frac{\tau_I}{\tau_U} \right) \\
0 & \text{otherwise} 
\end{cases} (22)
\]

The aggregate demand for information is obtained by integrating among the traders, as is equal to
\[ \iota^A = \begin{cases} 
1 & \text{if } c < \frac{1}{2\gamma} \ln \left( \frac{\tau^I}{\tau^U} \right) \\
\lambda & \text{if } c = \frac{1}{2\gamma} \ln \left( \frac{\tau^I}{\tau^U} \right) \\
0 & \text{otherwise} 
\end{cases} \] (23)

Figure (9) plots the aggregate demand for information. The relative precision of informed and uninformed trader is a key component of the downward monotonic demand for information. The blue depicts the ratio of the posterior precisions of both types of traders, conditional on the disclosure of the manager, \( \delta = 1 \). The red-dashed line represents the demand for information conditional on a non-disclosure policy of the manager, \( \delta = 0 \). The strategic complementarity of information is captured by the fact that the traders value private signal, the lower is the aggregate demand for information. For lower values of \( \lambda \), the signal-to-noise ratio of the public signal \( z \) is relatively low, which increases the wedge of the precisions and the price traders are willing to pay for the private signal.

### 2.5.3 Trader’s Posterior Beliefs

The trader beliefs about the mean of \( \theta \), captured by \( \mathcal{M} \), are updated upon the observation of the manager decision. In particular, \( \pi = \Pr(\mathcal{M} = \mu \mid \delta = 1) \) denotes the posterior probability of the conditional mean being equal to \( \mu \), after observing the disclosure of the manager, which is given by

\[ \pi = \Pr(\mathcal{M} = \mu \mid \delta = 1) = \frac{\kappa 1/2}{\kappa (1 - \nu) + \kappa \nu / 2} \] (24)

Figure (10) shows the posterior probability about the conditional mean being equal to \( \mu \), for a trader that observed the manager disclosing. The posterior probability is increasing in the unconditional probability that the manager is privately informed, \( \nu \).
Figure 8: Asset Price Function.

Note: The figure displays the asset price function for different realizations of the proportion of informed traders. The red-dashed line displays the price function conditional on the disclosure of the manager, and a higher realization of the precision of the private signal: \( \tau = \tau_s + \tau_f \). The blue line the price function conditional on the non-disclosure of the manager, and a higher realization of the precision of the private signal: \( \tau = \tau_s \).

Figure 9: Aggregate Demand for Information

Uncertainty is measured by the conditional variance of the liquidation value of the asset \( \theta \). The econometrician uncertainty is captured by the conditional variance of the uninformed trader: \( \tau^{U-1} = \text{var}(\theta | I^U) \). For lower value of \( \lambda \), the relative precision of the informed and the relative uncertainty of the uninformed increases.
2.5.4 Information Producer’s Problem

Now turn to the information producers’ problem. Because information producers are ex-ante identical, it is assumed that each one receives the same information demand. The information producer chooses whether to pay the fixed cost \( c(\lambda) \) to discover \( \theta \), in order to sell signals at a competitive price \( c(\lambda) \). Information suppliers maximizes profits according to equation (5). Moreover, according to the free-entry/zero-profit condition, the price of information has to be such that the following condition holds \( \lambda c(\lambda) - \chi = 0 \). The aggregate information supply is

\[
v^S = \begin{cases} 
\infty & \text{if } c(\lambda) > \frac{\chi}{\lambda} \\
(0, \infty) & \text{if } c(\lambda) = \frac{\chi}{\lambda} \\
0 & \text{otherwise} 
\end{cases}
\]  

(25)

Assumption 1. The fixed cost of information discovery is strictly higher than the price traders are willing to pay is every other trader is informed \((\lambda = 1)\),

\[
\chi > \frac{1}{2\gamma} \ln \left( \frac{\tau_\theta + a^2\tau_u + \tau_\epsilon}{\tau_\theta + a^2\tau_u} \right)
\]

(26)

This assumption is imposed to simplify the analysis by concentrating in the information market equilibria where a proportion strictly less than 1 becomes informed, and so avoiding multiple equilibria.

Assumption 2. The fixed cost of information discovery is such that the average cost of information is strictly lower than the price traders are willing to pay at the tangency point,

\[
\chi < \frac{1}{2\gamma} \left( \frac{\tau^U}{(\tau I)^2} (a^2 \lambda^2 \tau_u \tau_\epsilon) \right)
\]

(27)

This assumption is imposed to make sure that there always exist an interior solution in the information market, which determines the existence of an equilibrium with strictly positive information supply.

The following Figure (11) displays the information market equilibrium. Given the presence of
increasing returns to scale, information supply is a decreasing function of $\lambda$. The figure shows 3 possible equilibria. Point A is the less interesting solution in which there is no provision of information and the price of it is $c = \infty$. Given assumptions (1) and (2), there always exist an interior solution, so point A will be omitted from the analysis. Point B is and stable solution, since information producers have incentives to provided more information at a price higher than the average cost. Point C is the stable interior solution, and will be the focus of the analysis.

**Proposition 3.** Given the optimal disclosure decision of the firm’s manager, the optimal traders’ asset demands $x_i(s_i, p)$ and $x_i(p)$, the equilibrium asset price $p(\lambda)$ and assumptions (1) and (2), there are 3 equilibria in the information market: one equilibrium with zero information supply (point A), one unstable equilibrium with strictly positive information supply (point B) and one stable equilibrium with strictly positive information supply (point C).

**Proof.** The equilibrium with zero information supply is obtain when the information producer charge a price $c = \infty$ at which situation information demand is $\iota^A = 0$. The equilibria with strictly positive information supply are obtained by solving the following condition:

$$\frac{\chi}{\lambda} = \frac{1}{2\gamma} \ln \left( \frac{\tau_\theta + \tau_u \lambda^2 + \tau_e}{\tau_\theta + \tau_u \lambda^2} \right)$$

(28)

Given assumption (1), the solution to the previous equation is bounded above $\lambda^* \leq 1$. In this way, assumption (2) assures there exists at least one equilibrium with $\lambda^* > 0$. 

**Corollary 1.** Total volatility $\text{var}(p) + \text{var}(\theta | p)$ is constant, but depends negatively on $\lambda$.

**Proof.**

$$\text{var}(p) = \text{var} \left( \frac{z(\rho + \lambda \tau_u \rho^{-1}) + \tau_\theta \theta}{\lambda \tau^I + (1 - \lambda) \tau^U} \right) = \text{var} \left( \frac{z(\rho + \lambda \tau_u \rho^{-1})}{\lambda \tau^I + (1 - \lambda) \tau^U} \right)$$

$$= \text{var}(z) \left( \frac{\rho + \lambda \tau_u \rho^{-1}}{\lambda \tau^I + (1 - \lambda) \tau^U} \right)^2 = \left( \frac{\rho + \lambda \tau_u \rho^{-1}}{\lambda \tau^I + (1 - \lambda) \tau^U} \right)^2 \left( \frac{\rho + \lambda \tau_u \rho^{-1}}{\lambda \tau^I + (1 - \lambda) \tau^U} \right)^2$$
Figure 10: Posterior Probability about the conditional mean of $\theta$.

Note: The conditional mean of $\theta$ is given by $\mathcal{M}$, which can take two values, either $\mu$ or $-\mu$.

Figure 11: Market for Financial Information.
\[
\text{cov}(\theta, p) = \text{cov}(\theta, \left( \frac{z(\rho + \lambda \tau \epsilon \rho^{-1}) + \tau \bar{\theta}}{\lambda \tau t + (1 - \lambda) \tau U} \right)) \\
= \text{cov}(\theta, z) \left( \frac{\rho + \lambda \tau \epsilon \rho^{-1}}{\lambda \tau t + (1 - \lambda) \tau U} \right) = \left( \frac{a \lambda}{\tau} \right) \left( \frac{\rho + \lambda \tau \epsilon \rho^{-1}}{\lambda \tau t + (1 - \lambda) \tau U} \right)
\]

\[
\text{var}(\theta | p) = \text{var}(\theta | p) = \frac{1}{\tau U}
\]

(29)

In equilibrium, the informed trader \(i\) buys or sells the asset according to whether her private estimate of \(\theta\), is larger or smaller than the market estimate, \(p\). The informed trader trade more intensely (higher \(a\)) if the precision of the signals (\(\tau_\epsilon\)) is higher or risk aversion (\(\gamma\)) is lower.

### 2.6 Manager’s Problem

Consider the manager’s optimal disclosure decision. As shown in figure (7), and prove in lemma (3) below, \(\lambda\) takes one of two possible values in the stable equilibrium: \(\lambda \in \{\lambda_-, \lambda_+\}\), depending on the precision of the private signal, \(\tau_\epsilon\). Denote by \(p(z, \lambda, \delta)\) the price function according to (10) when the proportion of informed traders is given by \(\lambda\). In particular, when the market price is \(p(z, \lambda, \delta = 1)\), informed traders put more weight on their highly precise idiosyncratic signal, allowing a more precise public signal \(z\).

Given the linearity of the asset prices, the optimal disclosure policy of the manager is described by a threshold strategy similar to Acharya et al. [2011].

**Lemma 2.** Given the asset pricing rule (10), where \(A(\lambda)\) and \(B(\lambda)\) are monotonically decreasing and increasing in \(\lambda\) respectively, and the equilibrium behavior for \(\lambda\), the manager discloses his signal to the information producers whenever he is privately informed and \(\mathcal{M} = \mu\).

**Proof.** Given the pricing rule (10) and \(\lambda\), the manager’s price expectations are linear on \(s_f\):
\[
E(p | s_f) = A(\lambda) + B(\lambda)E(\theta | s_f).
\]
In particular, \(s_f\) is such that the manager is indifferent between disclosing the signal, or concealing the information in which case investors can not distinguished whether is manager is informed or not. Equation (30) shows the manager’s conditional expected price when he discloses and when he conceals, in the LHS and RHS respectively.

36
\[ A(\bar{\lambda}) + B(\bar{\lambda})E(\theta \mid \tilde{s}_f) = A(\bar{\lambda}) + B(\bar{\lambda})E(\theta \mid \tilde{s}_f) \] (30)

Using the result that \( E(\theta \mid s_f) = (\tau_\theta \bar{\theta} + \tau_{sf} \bar{s}_f) / (\tau_\theta + \tau_{sf}) \), the previous inequality delivers the result. In particular, proposition (2) shows that \( B(\bar{\lambda}) > B(\bar{\lambda}) \).

**Proposition 4.** There exists an equilibria composed by solutions considered in proposition (2) and (3) and beliefs given by (24) where priors are given by

\[
\bar{\theta} = \begin{cases} 
E(\theta \mid s^F \geq \tilde{s}^F) & \text{if there is disclosure} \\
(1 - \kappa)E(\theta) + \kappa(1 - \nu)E(\theta \mid s^F < \tilde{s}^F) & \text{if there is no disclosure}
\end{cases}
\] (31)

and

\[
\sigma_\theta = \begin{cases} 
\text{var}(\theta \mid s^F \geq \tilde{s}^F) & \text{if there is disclosure} \\
(1 - \kappa)^2\text{var}(\theta) + (\kappa(1 - \nu))^2\text{var}(\theta \mid s^F < \tilde{s}^F) & \text{if there is no disclosure}
\end{cases}
\] (32)

**Lemma 3.** Following the result from proposition (3) and the two possible realizations of \( \tau_\epsilon \) given by equation (4), there two equilibria in the information market with strictly positive information provision: \( \{\bar{\lambda}, \bar{\lambda}\} \). Higher values of \( \tau_\epsilon \) are related with \( \bar{\lambda} \).

**Proof.** Results follow from the proof of proposition (3).

### 3 Conclusion

While the role of uncertainty in the financial decision making is crucial, economist are still trying for identify the drivers of uncertainty. This paper presents a model where the informational frictions are key. The model has two main components. First, the disclosure decision of the firm’s manager about the future dividend payments. Second, the investors’ information acquisition decisions in a
market for information where producers face increasing returns to scale. The novel aspect of the model is the interaction between the signalling problem of the manager when deciding whether to disclose information over the business cycle, and the incentives for investors to acquire high precision information.

This article presents a dynamic rational expectation equilibrium of an overlapping generation model of asset prices with heterogeneous private information. Investors can actively learn about the future dividend payoff by acquiring information, in the spirit of Grossman and Stiglitz [1980]. Following Veldkamp [2006], we introduce a competitive market for information with free entry, where information producers face increasing returns. This is key for the result. The first order condition that determines the optimal information demand require the marginal benefit of information acquisition to be equal to the marginal cost. In equilibrium, the marginal benefit is given by the relative precision of the informed to the uninformed investor. As long as the marginal cost is constant, the relative precision of investor’s beliefs will also be.

Finally, following Acharya et al. [2011] we consider the strategic decision of the firm’s manager of information disclosure. The manager might be privately informed, and decides whether to disclose his private signal to the information producers to improve the precision of the signals offered in the information market. Given that investors don’t know whether the manager is informed, he can conceal his private signal and pretend to be uninformed. The manager follows a threshold strategy and discloses his signal only it is above a certain threshold.

The moment conditions of the representative investor are weighted using the proportion of types of investors in the market, which fluctuates during the business cycle. In recession, the conditional moments of the uninformed trader are more heavily represented, while the opposite is true during expansions. The model amplifies the excess return paid over the risk-free rate, introducing an alternative explanation for the equity risk premium.

4 Appendix
4.1 Measuring Uncertainty

This section presents survey-based measure of uncertainty. Although uncertainty can take many forms and sources of uncertainty are diverse, the focus in this paper is on the uncertainty of financial agents about the cash flows of the risky asset. Despite the concentration in one specific source of uncertainty, measuring uncertainty poses many difficulties. Reasonably high-quality output price data is not available on a sufficiently disaggregated basis, and technology shocks are largely unobservable. Moreover, uncertainty concerns not what actually happens but what might occur, and data on expectations are notoriously poor.

In this section, I present study the behavior of two of the most widely used measures of uncertainty. The first measure is the stock-market volatility has been previously used as a proxy for uncertainty by Leahy and Whited [1996], Bloom, Bond, and Van Reenen [2007] and Bloom [2009]. The second is the survey-based measure of uncertainty, the cross-sectional dispersion of forecasters’ predictions used by Leduc and Sill [2010] and Mankiw, Reis, and Wolfers [2003], calculated using the quarterly Survey of Professional Forecasters from the Philadelphia Federal Reserve.

4.2 Implied Volatility Index (VXO)

Empirical tests for uncertainty are not immediate because there is not an available measure of the time series uncertainty of investors. The VXO index is calculated by the Chicago Board Options Exchange using prices for a range of options on the S&P 100 index. It was first published in 1993, and since 2006, options contracts on the VXO became available. The VXO represents the square-root of the risk neutral expectation of the S&P 100 annualized variance over the next 30 calendar days.

Figure (12) depicts the evolution of the VXO, and reflects fairly clear that the stock-market volatility increases during recession. However, recession has a poor explanatory power over the VXO. The results of a linear regression of the stock-market volatility on a constant plus a dummy variable indicating NBER recession periods for the period 1960-2013 displays an $R^2 = 0.16$. Including several macroeconomic variables such as industrial production, employment, hired hours, inflation, wages and the fed funds rate provide a very small explanatory power of stock-market
4.3 Survey-Based Measures of Uncertainty

The focus of this paper is on the survey-based measure of uncertainty. This measure is constructed using the quarterly Survey of Professional Forecasters from the Philadelphia Fed., and considers the cross-sectional dispersion of beliefs among forecasters. A higher dispersion of beliefs reflects a higher uncertainty about future realizations of the fundamentals of the economy. Data is available since 1968, but the first three years are discharged due to a high proportion of missing entries, restricting the sample from 1971q1 to 2011q1.

At time \( t \), forecasters submit predictions for \( \tau \) quarters ahead, where \( \tau \in \{-1, 0, 1, ..., 4\} \) and \( \tau = -1 \) indicates forecast for the last publicly know observation of the variable, and \( \tau = 0 \) indicates forecast of the realization the current quarter, which typically ends 6 weeks after the deadline to
submit the questionnaire. The number of forecasters range from 75 to 9 (for the exceptional quarter of 1990q2), and the mean of forecasters is 37. In order to increase the number of forecasters’ reports, we consider predictions for the following variables: industrial production, corporate earnings, inflation and nominal GDP.

This database has two interesting features. First, forecasters are anonymously identify in the panel with an ID number. While it is possible to follow a particular forecaster over time, it is not possible to identify him with a real identity. This prevents forecasters from playing strategically when submitting the forecast, avoiding the forecasting the forecast of others type of issue and truth-telling reporting.\footnote{This problem has been widely study in the literature, beginning with Jovanovic [1982] and Townsend [1983]}

Second, it is possible to classifying the forecasters by industry. The industry indicator distinguishes financial service or non-financial service forecasters providers, or other (unreported industry). This suggests a potential classification for the sample. Informed forecasters are assumed to be those from financial service providers, while uninformed forecasters are represented by the complement. The argument relies in the fact that financial service providers usually pay for information services and are up-to-date with respect to the available information in the economy, carrying a research department by their own. The research provided by this department will be useful in estimating the cash flows of risky securities forehand.

Specifically, for each quarter $t$, let $f_i(t, \tau)$ be the forecast of agent $i$ of the of the level variable $a$ of interest at time $t+\tau$ where $\tau$ is forecasting the horizon, and let $a(t)$ be its last publicly know level of the variable. If $n_t$ is the number of individuals at time $t$, we then define the time $t$ dispersion of beliefs on the variable $a$ at time $t+\tau$ as

$$d_a(t, \tau) = \sqrt{\frac{1}{n_t - 1} \sum_{i=1}^{n_t} \left( \frac{f_i(t, \tau)}{a(t)} - \frac{1}{n_t} \sum_{i=1}^{n_t} \frac{f_i(t, \tau)}{a(t)} \right)^2}$$

(33)

Consider the measure of uncertainty at time $t$ about the future $t+\tau, d(t, \tau)$, as the sum of the dispersion of beliefs for the variables $a$, where $a$ indicates industrial production, corporate
earnings, inflation and nominal GDP,

\[ d(t, \tau) = \sum_a d_a(t, \tau) \]  \hspace{1cm} (34)
References


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